# The audibility of the temporal characteristics of audio systems

Dr. Hans R.E. van Maanen

(www.temporalcoherence.nl info@temporalcoherence.nl)

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**ABSTRACT**

The characteristics of audio equipment are rarely studied in time domain, mostly in frequency domain. Although the characteristics in frequency domain are important, the primary way humans here sound is in time domain. The characteristics of an audio system in time domain should therefore be given more attention. In this paper, some common misunderstandings of the Fourier theory are discussed, explaining why the attention has shifted too much towards the frequency domain. The misunderstandings are illustrated by some paradoxes and these are used to direct the developments towards improved temporal response, based on the properties of human hearing. The improvements, which can be obtained, are illustrated with the application to pick-up cartridges, the CD-filtering, cross-over filtering and compensation of a woofer in an acoustic box, both in frequency and time response.

The application of this approach has resulted in audio systems, which are judged very high in natural sound by objective listening tests by many individuals.

In the appendices, the basic calculation procedures to determine the temporal response of systems are presented, including the source code of a QuickBasic program to perform such calculations.

## 1. Introduction.

The reproduction of sound has been a challenge since Edison invented the gramophone. Still, no "perfect" solution has been found and every audio system is a compromise. On top of that, no unique procedure for the measurement of audio systems has been found which provides an objective measure of its quality with respect to the human ear[1]. So in order to find the optimum compromise, a lot of "trial and error" is applied, which, unfortunately, leads to a large amount of work without a guarantee that the result will be an improvement. Surprisingly, through the years, most attention has been given to the behaviour of audio systems in *frequency domain* and to the non-linearities (distortions), often in the frequency domain as well. The *temporal behaviour* of audio systems has been neglected (or ignored), at best limited to the response to composite signals like square waves, probably because of the theory behind the Fourier Transformation, which states that the information in time and frequency domains is identical. However, that is only true to a limited extent and, ignoring the limitations, can lead to erroneous conclusions, as will be the subject of sec. 2. In sec. 3, we will discuss several examples of the temporal behaviour of audio systems and how these can be improved. This will lead us to the conclusions, laid down in sec. 4.

## 2. The relation between time and frequency.

Many audio systems have specifications in the *frequency domain*, like the frequency response. Most of us are aware of the concepts of "high" and "low" frequencies. One could wonder why audio systems are specified in frequency domain and what it tells us. The background is a rather complex and complicated mathematical theory, of which the foundations were laid by Jean Baptiste Fourier in the early 19th century (ref. 1 - 3). In general, the theory is about "transforming" a function of "*x*" into a function of "1/*x*". It tells us that it is possible to perform such a transformation on a "one-to-one" basis. This means that every function of "*x*" has only one corresponding function of "1/*x*" and that no other function of "*x*" has the same corresponding function of "1/*x*", which means that also the reverse is true. In mathematical terms, the function of "*x*" and its transform (the function of "1/*x*") are therefore identical, albeit that they look completely different for ordinary human beings. The most common application of this theory (but certainly not the only one) is the transformation from "*t*" (time) to "1/*t*" (frequency). So does this mean that we can specify an audio system solely in frequency terms? The answer is: not completely and it depends on the amount of information which is preserved. This is not trivial and therefore, I will explain this further below.

### *2.1 Fourier analysis applied to determine non-linearity of systems.*

One of the more well-known applications of the Fourier theory is that it enables one to write any[2] periodic signal as a (basically infinite) series of sines and cosines. This is often applied to determine the harmonic distortion of systems[3]: a pure sine wave is fed into the system and the output also contains harmonics of the input frequency, which are integer multiples of the input frequency, as is illustrated in fig. 1. By presenting the distortion in this way, one can see which harmonics contribute to what extent and -hopefully- guesstimate what its effect will be on the perceived sound quality. This, however, is not an easy question to answer. In general, solid state amplifiers generate far less harmonic distortion than e.g. loudspeakers and valve amplifiers. Yet, the distortion from solid state amplifiers is regarded as far more annoying and tiring than the distortion of loudspeakers and valve amplifiers. This could be related to the composition of the harmonic distortion: whereas solid state amplifiers can

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[1] With the "human ear" I certainly not only mean the organ itself, but also the supercomputer in the brain which processes the signals!

[2] Of course, mathematicians have invented functions which cannot be transformed, but *all* sound signals, periodic and non-periodic, can be transformed.

[3] Systems can basically mean anything with an input and an output. Examples are an amplifier and a loudspeaker box, even though the output of the latter is sound.

generate very high harmonics (up to over the $30^{th}$), loudspeakers and valve amplifiers are commonly limited to the $5^{th}$ harmonic, which also occur in "natural" instruments. Therefore, a presentation like in fig. 1 provides more information than a single distortion number, albeit that still the audibility and severity is hard to estimate. However, we will not dig deeper into this as it is not the main topic of this report.

### *2.2 More general application of the Fourier Transformation.*

The Fourier Transformation is far more powerful than just the separation of the different harmonics to a distortion measurement. One very interesting application is that it allows the calculation of the response of a system to an arbitrary signal, including discontinuous signals. This, however, leads to some complex and complicated mathematics, which, fortunately, can nowadays easily be run on a PC or laptop. But the Fourier Transformation leads -in general- to a function in frequency domain which is complex (which means that it has an imaginary part), even if the signal in time is real (like e.g. the signal from a microphone or the analog output of a player). To make life a little easier, such a complex signal is often presented as an *amplitude* and a *phase* spectrum[4]. And this is where the first problem sneaks in: for us, human beings, the amplitude spectrum is relatively easy to grasp (see e.g. fig. 1), but the phase spectrum is more difficult to comprehend. So we are tempted to forget about the phase spectrum, but in order to keep the one-to-one transformation, we need *both*. This will be illustrated by some examples.

#### 2.2.1. Signals with the same amplitude spectrum but different time behaviour.

It is not hard to prove that signals with the same amplitude spectrum, but with a different time behaviour, exist. The "recipe" to generate such signals is presented in Appendix A. The signals, shown in figs. 2 and 3 are markedly different, yet, as can be seen from figs. 4 and 5, there amplitude spectra are identical. We can therefore already conclude from these graphs that phase errors "translate" into changes in the temporal behaviour of a system and that it can (but not necessarily has to) lead to time smear (compare signal #2 with signal #1). This shows that the elimination of the phase also eliminates the uniqueness (the one-to-one relation) of the Fourier Transformation.

The concept of a "transformation" seems rather mysterious to many people, but -surprisingly- we apply it almost daily. We all know that the Earth is a more or less spherical celestial body. To find our way on it, however, we use flat, rectangular sheets of paper with horizontal and vertical lines drawn on it (named latitude and longitude) and we pinpoint every spot on the Earth uniquely on this sheet of paper. However, to find a place back, we need both latitude *and* longitude. If we only provide latitude, the place we are looking for is still included, but it may be way out from where we are looking. So this is similar to our signals: if we replace "latitude" by "amplitude" and "longitude" by "phase", we have the same situation: leave one out and there are many different signals which have the same amplitude spectrum.

#### 2.2.2. Some additional properties of the Fourier Transformation.

Before we will start a discussion on the way we hear, we need to mention that an important property of the Fourier Transformation is that it is a *linear* transform. This means that application to non-linear systems should be done with caution and that its results have to be interpreted with care.

An important result of the theory behind the Fourier Transformation leads to the "Fourier Inversion Theorem". In heuristic terms, it states that "things which are fast in time domain are slow in frequency domain and vice-versa". As the independent variables are each others reciprocals ("*t*" and "1/*t*"), this is not really surprising. The mathematical expression of this theorem is:

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[4] It is impossible to present the theory of complex numbers in this report. Numerous textbooks are available on this subject and the interested reader should be able to find it easily. Undergraduate textbooks usually are the best entrance to this subject.

$$\Delta t \,.\, \Delta f \geq 2\pi$$

in which:

$\Delta t$ = width in time domain s

$\Delta f$ = width in frequency domain Hz

This theorem has severe consequences, as we will see later. To illustrate it, the impulse responses of a 4th and a 12th order Butterworth low-pass filter are presented in figs. 6 and 7, illustrating the theorem. These examples are analog filters with no linear phase characteristics, but that does not matter: the Fourier Inversion Theorem is valid for *all* systems, analog and digital, phase linear or not.

### *2.3 The way we hear.*

It is probably banging on an open door, but we hear sound primarily as a function of time. ...sdrawkcab cisum fo eceip a yalp dluohs decnivnoc ton era ohw esohT[5]
(**N.B.** The amplitude spectrum of a piece of music played backwards is *perfectly identical* to that of the piece played normally!)

A more convincing example is probably when we would be able to delay the fundamental of a stroke on a kettledrum by one second, compared to the upper harmonics. It is obvious that this will be audible. So the question is not *if*, but *to what extent* and *in what way* our ears are sensitive to the temporal properties of sound.

As "phase" in frequency domain is clearly related to the temporal behaviour of signals, it is reasonable to assume that our ears will be sensitive to phase. But, paradoxically, this is not the case with continuous sine waves. Which let to the conclusion by some investigators that the human ear is *not* sensitive to phase. But this is, however, jumping to conclusions and this often leads to incorrect results.

The human ear distorts the incoming sound by some 25% (!). This lead a professor in Leiden (Netherlands) to state that the design and building of amplifiers with a distortion figure of less than 1% was absolutely useless. Probably the man knows a lot about our hearing, but has not used his own ears.... Nature is usually efficient: things which are not needed disappear quickly due to evolution as it is better not to waste energy on things one does not need. Therefore, it is reasonable to assume that we use this distortion to get more information about the sound than would be possible by a linear system. And this could possibly be a (partial) explanation of the phenomena: non-linear systems also are able to detect the *envelope* of a signal. And the envelope is a distinct difference between the signals #1 (fig. 2) and #2 (fig. 3). Whereas the envelope of signal #1 is a rectangle[6], the envelope of signal #2 gradually increases and decreases in amplitude, as is illustrated in fig. 8. The interesting thing is that the amplitude spectra of signal #1 and #2 are *identical* using *linear analysis*, but differ from the linear spectrum and *are different from each other* in the non-linear case. This is illustrated in figs. 9 and 10, compare with figs. 4 and 5. So in this way, ours ears could be *indirectly* sensitive to phase; note that the envelope of continuous sine waves does *not* change when the phase between the two is changed! Also note that e.g. a spectrum analyzer based on the (linear!) Fourier Transformation (and basically all are), the result is not necessarily representative for our ears. So its output will have to be interpreted with care.

If our ears are able to detect the envelope of signals and uses this information, this has consequences for the design of audio-systems:

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[5]Those who are not convinced should play a piece of music backwards....

[6] This is easy to see as signal #1 can be obtained by multiplying a rectangle with a continuous sine wave: where the rectangle is "1", the cycles remain, where it is "0", no signal is left.

- Frequencies above the sine wave limit[7] can contribute to the perceived sound.
- The sine wave response provides incomplete and thus insufficient information.
- Cross-over filters need to be designed for correct temporal response.
- The use of resonances to create "linear" frequency response should be outlawed.
- Most important: the overall temporal response of audio systems should be optimized.

We will now discuss the above aspects in more detail.

In his famous book "Loudspeakers", G.A. Briggs (ref. 4) describes an experiment in which two elderly persons were tested: one was stone deaf above 10 kHz, the other above 11 kHz. But both could unambiguously tell when a 12.5 kHz low pass filter was inserted or not. A similar result will be discussed in the examples in the next section.

Because the sine wave response only gives information, related to the amplitude spectrum of the response, it is impossible to determine the properties of the response in time domain. The information is therefore incomplete. However, one is easily misled by the information.

Cross-over filters are required because loudspeaker units are only able to handle a part of the audible frequency range. The requirements, however, are conflicting and can easily give rise to designs which work well in the frequency domain, but which are disastrous in the time domain. This will be shown in detail in the next section.

The use of resonances to create "linear" frequency response curves looks attractive, because they usually are very efficient. However, resonances destroy the temporal response and are therefore very undesirable from a sound reproduction point of view. This will be illustrated with the time response of woofer systems in the next section.

In my view, the temporal response characteristics of audio systems should be optimized in order to create a natural sounding system. The perceived quality of e.g. electrostatic loudspeakers is probably at least partly due to the good temporal response characteristics of such loudspeakers. In the next section, some examples of improvement of the temporal response, which I have encountered through the years of my own developments, will be discussed.

## 3. Examples of temporal response improvements.

Through the years, I have encountered a number of examples, in which the temporal response characteristics play a role. Although not complete, the three most important ones are the response of moving magnet (MM) pick-up cartridges, cross-over filters and woofer systems. These will be discussed in detail below.

### *3.1 Moving magnet pick-up cartridges.*

Before the CD became popular, the most commonly used consumer medium was the long-play (LP) record. The two most important types of pick-up cartridges for HiFi applications were the moving magnet (MM) and the moving coil (MC) cartridges. As could be expected, these were tested by audio-magazines and surprisingly, usually the MM gave better results on the *metered* part of the test, but when it came to *listening*, MC's were clearly superior. Although the why of this was often wiped under the carpet by stating "our ears behave different than a meter", it is unsatisfactory to have such a strong contradiction. There is, however, an essential distinction between the MM and the MC cartridges: the induction coil of the MM is (far) larger than those of MC elements. This is not so hard to understand, as the induction coil of MC's has to be moved by the needle, whereas those of MM's do not move. So in order to get more signal, the coil induction of MM's is made large (0.5 - 1 H), but that creates, in combination with the capacitance of the cable between the cartridge and the input impedance of the

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[7] Usually set at 20 kHz.

pre-amplifier, a low-pass filter in the range of 8 - 12 kHz. Of course, such a response is unacceptable and in order to increase the frequency response to roughly 20 kHz, use is made of a mechanical resonance of the magnet on the bar, connected to the needle. The resonance frequency normally lies in the range of 18 - 19 kHz and has a height, depending on the electrical properties of the cartridge, of 6 - 10 dB. The resulting frequency response curve looks like something, shown in fig. 11, in which also the contributions of the electrical and mechanical part are shown separately. Although this creates a frequency response curve, which fulfils the normal requirements for HiFi systems, the impulse response of such a system is far from ideal. This is illustrated in fig. 12, where the temporal response to a complex signal is drawn. The output signal looks more like a lower-frequency tone burst and shows significant time-smear. On the other hand, the MC cartridges do not have this electrical low-pass filtering in the audio band and in order to avoid coloration, the mechanical resonance occurs at far higher frequencies. As a rule of thumb, MC cartridges have a frequency response up to 80 kHz. It is therefore to be expected that these will reproduce such a complex signal far better in time. This is illustrated in fig. 13 and this severe difference indicates that the *perceived* difference in quality between MM and MC is caused by the temporal response characteristics.

The easiest way to solve the problem is to discard the MM cartridge and to change over to a MC cartridge. However, MC cartridges have a number of disadvantages, like the small signal strength, which requires either a pre-pre-amplifier or a step-up transformer, and the needle replacement, which has to be done in the factory. So I chose to take a different approach: an electronic compensation of the mechanical resonance of the cartridge. Although this is rather risky because of stability of the compensator, I managed to create a design which is able to do this. The details are described in two publications (in Dutch, ref. 5 and 6). The improvement obtained is shown in fig. 14 and although this is not equivalent to the MC response, the improvement is obvious when compared to the response, shown in fig. 12. But the question, of course, is whether this is audible or not.....

When I listened to the first prototype, I was amazed, even though the compensator had some flaws. After solving the teething problems, I did not believe my own ears, so I arranged for a listening panel to judge the outcome. Five people were in the panel, two experienced listeners, two unexperienced listeners and one averaged experienced listener. We arranged for two identical turntables with identical cartridges, one operated as prescribed by the manufacturer and one via the compensation processor. The test was an A-B comparison by playing two copies of the same record, only I knew which one we were listening to. In thirty *seconds*, the whole panel agreed that there was a clear difference in the perceived quality of the sound and they all agreed that the sound from the compensation processor was obviously superior. So we are not talking about subtle effects. It seems that some manufacturers knew about these problems as a few MM cartridges were brought to the market with a higher frequency of the mechanical resonance and these were indeed classified as superior to "normal" MM cartridges, albeit not as good as MC cartridges. We tried the compensator on such a cartridge and perceived an increase in the quality of the reproduction. This all is in full agreement with the above reasoning and results.

Based on this experience, I was able to predict that the strong anti-alias and reconstruction filtering of digital audio systems, running on sampling frequencies of 44 - 48 kHz, would introduce audible artifacts. If we just look at the reconstruction filtering of a CD player, an example of the impulse response of a linear phase version is shown in fig. 15, we see a large time smear, in agreement with the Fourier Inversion theorem. Such responses, including the step (square wave) response as shown in fig. 16, have been published in numerous audio journals, so these are probably familiar to you. But if you send the complex input signal through it, you obtain the response, shown in fig. 17. When you compare it with the response, shown in fig. 12, knowing this was clearly audible, it is not unrealistic to predict that this response will also be clearly audible. Back in those days, my comments were not highly appreciated (to put it diplomatically, "if looks could kill" is a more appropriate description), but it is now generally accepted. Actually, the manufacturers of the new consumer media, like the SACD[8], claim that the significant improvement of these systems is also because of the improved temporal response. This

[8] SACD = Super Audio Compact Disc

is illustrated by fig. 18, which was reproduced using ref. 7 and indeed, the temporal response of a SACD is incomparably better than that of a CD. So my claim in 1981 (ref. 8, in Dutch) that we would want a better digital standard in 10 years time did not completely come through, but regarding the moment the DSD[9] developments started, I was not that wrong......

Through the years, it was not difficult for me to demonstrate the superior quality of the LP compared to the CD. Even ordinary LP's gave far more detail in the upper regions than the CD, let alone when specials, like half-speed masters[10], were used. The sound, coming from the SACD is excellent and especially the ease of the sound is a relief. With CD's I (too) often sit with my toes cramped, with the SACD this does not happen.

### *3.2 Cross-over filters.*

Cross-over filters are needed because the loudspeaker units we use are not able to reproduce the whole audio range and start to behave undesirably when they have to respond to frequencies outside the range they can handle. With e.g. woofers this results in undesirable resonances in the cone, with tweeters, the low-frequency power can even demolish them. The cross-over filters thus should eliminate those frequencies the unit cannot handle. When the filters for e.g. a three-way system should be designed, a low-pass filter is needed for the woofer, a band-pass filter for the squawker and a high-pass filter for the tweeter. So at first sight, steep, high-order filters are attractive as these suppress the undesirable frequency ranges the best. But the flip side of the coin is that the temporal response characteristics of steep filters is long, which is a direct consequence of the Fourier Inversion Theorem. This creates a number of problems, which will be discussed using several different examples.

#### 3.2.1. Fourth order Butterworth filters.

To illustrate the extension of the response in time domain, the response of a $4^{th}$ order Butterworth low-pass filter on a two-cycle triangular toneburst is shown in fig. 19. The response of a $4^{th}$ order Butterworth high-pass filter is shown in fig. 20. The summed response of these filters is shown in fig. 21, which shows that the filter combination creates distortions in the temporal characteristic. These are so severe that these can no longer be corrected, especially the time smear is in this respect prohibitive. And as the cut-off frequencies of cross-over filter lie in the middle of the audible range, it is very likely that these effects are audible and reduce the perceived quality of the reproduced sound. Are there ways to avoid these problems?

#### 3.2.2. Fourth order Butterworth low-pass filter and subtraction.

An obvious approach is the use of subtraction, as is shown in fig. 22: there can be no doubt that the sum of the two outputs is identical to the input signal[11]. In fig. 23, the response of the $4^{th}$ order low-pass section is shown (cut-off frequency of 150 Hz) and -of course- the expectation is that the high-pass section will show a similar behaviour. But that expectation is incorrect, as is shown in fig. 24: the high-pass section is a weak filter with a large hump added to it, a little below the cut-off frequency of the low-pass section. The high-pass section behaves, ignoring the hump, as a first-order high-pass filter of 57 Hz as is shown in fig. 25. The hump of 5.15 dB at 130 Hz is required to compensate the

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[9] DSD = Direct Stream Digital

[10] "half-speed masters" are LP's which were created by using the original analog master tape at half the speed (16 2/3 rpm of the turntable) to reduce the deviations, introduced by the cutting head. Such LP's have a frequency response up to 50 kHz and good phase response up to 30 kHz. They usually sound superior to the "normal" LP version.

[11] Essential in the whole concept is that the subtraction is accurate, if not the suppression of the lower frequencies will suffer in an unacceptable way.

significant phase shift in the pass-band of the low-pass section, which is already 180$^o$ at the cut-off frequency. This means that the two loudspeakers units have to produce counteracting sounds, the largest part will have to end up as silence. This is illustrated by the responses, shown in fig. 26, in which the large amplitude of the high-pass section should be noted, in agreement with the hump, shown in figs. 24 and 25. Although it is possible, in theory, to create silence from two counteracting loudspeaker units, it will only work if the speaker units are perfect (but the main reason to use cross-over filters is that the units are *not* perfect!) and that the distances from the units to the listeners are identical. The latter can obviously not be true for all locations in the room, let alone the problems with reflections. The probability of coloration is therefore close to certainty. This is therefore not the optimum approach to solve this problem. Are digital filters the solution then?

### 3.2.3. Digital filters without phase shift.

The major advantage of digital filters is that *any* desired frequency response curve and phase characteristic can be created. So if we choose -as an example- a 6$^{th}$ order low-pass filter with no phase shift for all frequencies, the transfer function would look something like:

$$G_l = \frac{1}{1 + \omega^6\tau^6}$$

*in which:*

| | | | |
|---|---|---|---|
| $G_l$ | = | complex transfer function | |
| $\omega$ | = | radial frequency = 2 $\pi$ frequency (in Hertz) | 1/s |
| $\tau$ | = | time constant | s |

The high-pass complement would be:

$$G_h = 1 - G_l = 1 - \frac{1}{1 + \omega^6\tau^6}$$

$$= \frac{1 + \omega^6\tau^6}{1 + \omega^6\tau^6} - \frac{1}{1 + \omega^6\tau^6}$$

which yields:

$$= \frac{\omega^6\tau^6}{1 + \omega^6\tau^6}$$

As is quite obvious from the "symmetry" in the low- and high-pass sections, the filters are ideal from a frequency response curve point of view. So one is tempted to yell "Eureka", but is the response as ideal as one is initially thinking? Indeed, the most fundamental law of Physics, the law of Conservation of Misery, creates two major problems:

- Because of the zero phase shift, the filters show "pre-ringing", which means that they produce output signal before any input signal occurs[12], as can be seen in figs. 27 and 28. These figures also show the opposite responses of the low- and high-pass sections when there is no input signal.
- As a consequence, the sounds, coming from the two loudspeaker units, should cancel completely to create silence.

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[12] Of course, no system can predict the future, so the phenomenon is artificial. Processing in the digital domain usually means that the digital information is read from the mass storage device (e.g. a CD) and temporarily stored in a working memory. From this information, the output signal is derived, which always comes later than the storage of the digital information.

Although it is -in theory- possible to create complete silence by having two sounds cancel each other, it is in practice virtually impossible. First of all, the loudspeaker units should be perfect (which they are not, which is why we use cross-over filters in the first place!) and secondly, it can only work if the transit times from the loudspeaker units to the listener are precisely identical. With a physical separation between the loudspeaker units, this can never be true in the whole listening room and because of reflections, it is not likely to happen at any place in the listening room. So the situation is even worse than with the subtraction filter, the "pre-ringing" will very likely give rise to sounds before the actual sound starts, and such effects will not only be audible, but probably very annoying as this does not happen in reality.

### 3.2.4. Causal filters with a linear phase response.

Before I go to discuss the solution I have used, I would like to point out an elementary problem of *all* cross-over filters: the output signals of all the sections are *always* longer than the input signal. This is a direct consequence of the Fourier Inversion Theorem: as we reduce the width in frequency domain (which is the prime function of a filter!), we extend the width in time domain. Basically, there are three options:

- The extension occurs only àfter the end of the input signal.
- The extension occurs before the onset of the input signal ànd after the input signal has ended.
- The extension occurs only before the onset of the input signal.

The first is illustrated by fig. 26, the second by figs. 27 and 28. The third is theoretically possible with digital filters, but is so uncommon that we will ignore this. Note, however, that the second option can also show up in disguise: when linear phase filters are used, the whole response to the input signals can easily occur àfter the onset of the input signal as is shown in fig. 29. But a linear phase characteristic basically means nothing but a time delay:

$$\tau = -\frac{d\phi}{d\omega} = -\frac{1}{2\pi}\frac{d\phi}{df}$$

in which:

| | | | |
|---|---|---|---|
| $\tau$ | = | time delay | s |
| $\phi$ | = | phase angle | radians |
| $\omega$ | = | radial frequency = 2 $\pi$ frequency (in Hertz) | 1/s |
| $f$ | = | frequency | Hz |

So the *actual* response could still be before the *onset* of the delayed input signal! In that case, still two loudspeaker units have to make silence together as is illustrated in fig. 30. The only way to verify this is by calculating the temporal response of the filter sections and the combined response. Only then it is possible to see whether the filter combination acts like option 1 or option 2. In the case of fig. 30, it is obviously option 2 as the delayed input pulse appears àfter the filter sections have started to generate output.

### 3.2.5. Requirements for cross-over filters.

One could -correctly- state that when all the extension of the filter section output signals occur àfter the input signal has ended, it is also required that the two loudspeaker units create silence from counter-acting sounds. So why would this be more attractive than option 2? In my view, the following reasons make it a more attractive approach:

- The sounds, generated by "classical" (mechanical) instruments often have strong "attack", like a kettle drum or a piano, and decay slowly, compared to the attack. Therefore, the input signal does not stop abruptly, so the "silence" which needs to be created is at a far lower level.
- The room, in which the music is reproduced, has a reverberation time itself. Therefore, even if the sound would end abruptly, the sound pressure in the room would not. Therefore, even if there would be some sound generated by the loudspeaker units, it would not be heard easily.
- Although electronic instruments, like synthesizers, are able to generate signals which end abruptly (contrary to "classical" instruments), nobody knows how these sound in "reality", so if there would be some sound still coming from the loudspeaker units, nobody would notice.

Because of the above, cross-over filters should always be required to respond as in option 1.

### 3.2.6. Higher-order analog filters.

I have a phrase hanging above my bed: "*Filtering is a choice between different kinds of misery*". Optimisation is therefore to choose the misery which hurts the least. Regarding the above, the criteria for the filters I have used are:

- At least a second-order low-pass filter for the woofer to reduce the probability of cone resonances.
- At least a second-order high-pass filter for the tweeter to reduce the power handling requirements.
- The summed response of the filters should equal the input signal in time-domain.
- All the extension of the time signals should come after the input signal has ended (option 1, see above).

Therefore, I have chosen for analog filters, which can relatively easy be created in active systems. The original publication can be found as ref. 9, but as it is in Dutch, I will repeat here the derivation for the second-order low-pass filter and its high-pass complement, the derivation for the second-order high-pass filter and its low-pass complement is virtually identical and is left to the reader. The transfer function of a second-order low-pass filter with identical time constants is:

$$G_l(\omega) = \frac{1}{(1 + j\omega\tau)^2}$$

in which:

| | | | |
|---|---|---|---|
| $G_l$ | = | complex transfer function | |
| $\omega$ | = | radial frequency = 2 $\pi$ frequency (in Hertz) | 1/s |
| $j$ | = | complex operator ($j^2$ = -1) | |
| $\tau$ | = | time constant | s |

Its complement is:

$$G_h(\omega) = 1 - G_l(\omega) = 1 - \frac{1}{(1 + j\omega\tau)^2}$$

$$= \frac{(1 + j\omega\tau)^2}{(1 + j\omega\tau)^2} - \frac{1}{(1 + j\omega\tau)^2}$$

$$= \frac{(1 + 2j\omega\tau - \omega^2\tau^2) - 1}{(1 + j\omega\tau)^2}$$

$$= \frac{2j\omega\tau - \omega^2\tau^2}{(1 + j\omega\tau)^2}$$

$$= 2\,\frac{j\omega\tau\ (1 + j\omega\tau/2)}{(1 + j\omega\tau)^2}$$

which can be re-written as:

$$= 2 \frac{j\omega\tau}{1 + j\omega\tau} \frac{1 + j\omega\frac{\tau}{2}}{1 + j\omega\tau}$$

and thus consists of a constant, a first-order low-pass filter and a phase lag network. The corresponding characteristics are shown in fig. 31, using a time constant, corresponding to 300 Hz. This means that the low-pass filter has its -6 dB point at 300 Hz, its -3 dB point at 193 Hz. The phase-lag network creates a slight hump of 1.25 dB at 420 Hz in the high-pass section, which merely acts as a first-order high-pass filter. Although the price we have to pay for the second order low-pass filter is a slight hump, it is small, compared to the hump we obtained with the subtraction approach, using a $4^{th}$ order Butterworth low-pass filter: 1.25 vs. 5.15 dB. Slight imperfections of the units will therefore have far less detrimental effects.

The responses of the filter sections are shown in figs. 32, 33 and 34. As can be seen from these figures the filter sections fulfil the requirements as listed above while maintaining the correct overall temporal response. In my view, these kind of filters are the optimum choice for cross-over filters, regarding the current "state-of-the-art" in loudspeaker technology. The filters can easily be created for active systems, either by direct implementation of the time constants as prescribed from the equations (see also ref. 9) or obtained using the subtraction approach. The filters can, however, only be built approximately as passive sections in a "normal" cross-over filter.

I have applied these type of filters successfully for a large number of years and the high level of the perceived quality is -in my view- because of the correct temporal behaviour. The major disadvantage is that the slopes of the filters are only moderate and therefore, the behaviour of the loudspeaker units in their enclosure could be non-ideal, which is why I use electronic compensation for the units in their enclosure in order to get as close as possible to the correct temporal behaviour.

### *3.3 Low-frequency response of different loudspeaker enclosures.*

The reproduction of the lowest frequencies (say from 15 - 50 Hz) using cabinets which still have an acceptable size to put in a living room, has been a problem since the onset of sound reproduction. Although (folded) horns have been built, which use the walls of the living room as a part of the horn itself, this approach cannot be used in all living rooms, is even more problematic with stereo sound and suffers from the horn truncation. Therefore, we will not follow this road any further. The main three cabinet categories, which are used for low-frequency reproduction are:

- The bass-reflex cabinet.
- The transmission line cabinet.
- The acoustic box cabinet.

The bass-reflex uses both the resonances of the loudspeaker unit and the port to increase the output at low frequencies. This looks nice on paper, but a general comment about bass-reflex systems is that these sound "woolly" and "slow". This is not really surprising as resonances need time to grow and need time to decay, as is illustrated in fig. 35. As a result, the sound itself is delayed and the response to more impulse like signals is greatly distorted in time domain, as is illustrated in fig. 36, which shows the response of a bass-reflex system to a simulated kettle-drum signal. This is again shown in fig. 37, in which the envelope of the output signal is also presented. And although the kettle drum sound has a clear "attack", the reproduced sound has a clearly growing/decaying characteristic. So for sounds like organs, the bass-reflex may work reasonably well, for sounds which have a clear "attack", like a kettle drum or a grand piano, these are less well suited.

Transmission lines use a different approach: the loudspeaker unit is loaded with a (often a bit tapered) column of air, which ends in a port. The idea is that the port supports the loudspeaker unit below its resonance frequency (which is lowered due to the load of the column of air) at the anti-resonance of the column (= half the fundamental resonance frequency). The system deliberately uses no resonances and is therefore fundamentally better than the bass-reflex, but we should not forget the law of Conservation of Misery: the price one has to pay is that the column starts to act as a resonator (sort of pipe organ) at twice that frequency and to avoid this, the column should be damped by e.g. wool in such a way that it is fully "open" at half the fundamental resonance frequency and should be fully "dead" from the fundamental resonance frequency on. This is a constructive nightmare because it is very hard to predict how much damping material is required and "trial and error" requires numerous openings of the cabinet (in which the column is usually folded!), followed by listening tests and impedance measurements (as these reveal the resonances). It is not without a reason that Briggs qualified this design as "*a clumsy way to achieve a limited goal*" (ref. 4). So in practice, the construction of a transmission line is very complicated, time consuming and no guarantee can be given beforehand that it will work correctly, or even satisfactorily. All bass-reflex and transmission lines enclosures I have listened to through the years had some audible artifacts, which all reduced the pleasure of listening. This is why a different approach has been looked for.

The acoustic box is a well-known enclosure, which has the advantage of simple design and construction. When the volume of the enclosure is chosen well, the response is "critically damped", which means that the response is flat down to the resonance frequency and below it drops off with 12 dB/oct as shown in fig. 38. In general, the response is perceived as pleasant. The major disadvantage is that the air in the box acts as a spring, thereby increasing the resonance frequency of the unit (the suspension gets stiffer), which subsequently limits the low-frequency response.

The response of a loudspeaker unit in an acoustic box can be described by a damped linear mechanical resonator and is therefore similar to the mechanical resonance of the pick-up cartridge and can thus also be compensated in a similar way. I have Christened it "ABC-2", which stands for Acoustic Box Compensator, second order. The idea is to create an electronic simulation of the loudspeaker unit in its enclosure and use this simulation to give -a sort of- "feed forward" to correct the deviations. This would enable the extension of the response down to frequencies below the resonance frequency without compromising the temporal response.

Although the technique of the ABC-2 is feasible, as shown in fig. 39, there are some limitations:

- The amount of electric power required will increase as the efficiency of the unit will still decrease below the resonance frequency, this does not change[13]. The loudspeaker unit has to be able to handle this power.
- As the excursion of the loudspeaker cone is proportional to $1/f^2$, extending the low-frequency response will lead to unacceptably large cone excursions, which puts a limit to this approach[14].

Even with the above listed limitations, I have been able to achieve 16 Hz (-3 dB) from a 250 mm woofer in a 20 litre enclosure.

Because the actual response is compensated, the temporal behaviour of the system is also correct. This is illustrated in fig. 40 and 41, which shows the response to the simulated kettle drum of an ABC-2 compensated woofer (compare fig. 40 with fig. 36). The speed of the loudspeaker system is also illustrated by the time delay as a function of frequency, as determined by the Wigner distribution (ref. 10 and 11), shown in fig. 42.

## 4. Conclusions.

The temporal response of audio systems is given far less attention than the frequency response, even though our hearing acts in the time domain. The non-linear behaviour of our hearing is likely to

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[13] Note that the same problem occurs with Motional Feedback (MFB). Although the deviations of the response curve are corrected, the physical properties of the loudspeaker in its enclosure do not change and the increased response at lower frequencies needs to be realised using extra electrical power.

[14] Note that this is valid for any loudspeaker system. The reason is that force is proportional to the *acceleration* of the cone, which is the second derivative of the position. Constant Sound Pressure Level (SPL) thus corresponds to an amplitude of the cone excursion which is proportional to $1/f^2$.

be -at least partially- responsible for the indirect sensitivity to phase and thus the temporal response. Everything that I have done with my development work through the years which improved the temporal response of the audio system has had a positive effect on the perceived quality and natural sounding of the audio system. Because of these experiences, there is no doubt in my mind that the optimisation of the temporal response of audio systems is required for sound reproduction which sounds "natural".

The approach I have taken is to optimize the whole *system*, by integrating the "electronics" and the "mechanics" and by recognizing the limitations of the different components. Such a "holistic" approach can lead to a better *overall* performance and is -in my view- essential for a major improvement of the perceived quality.

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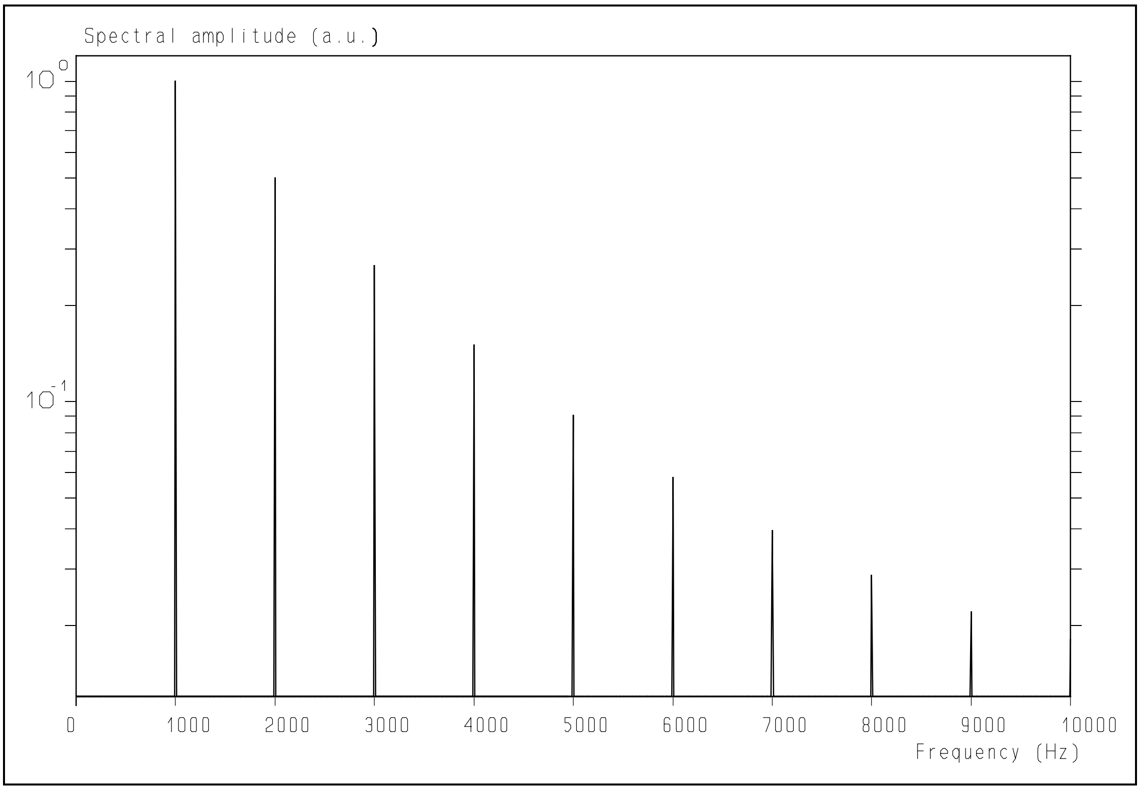

**Figure 1**: Example of harmonics.

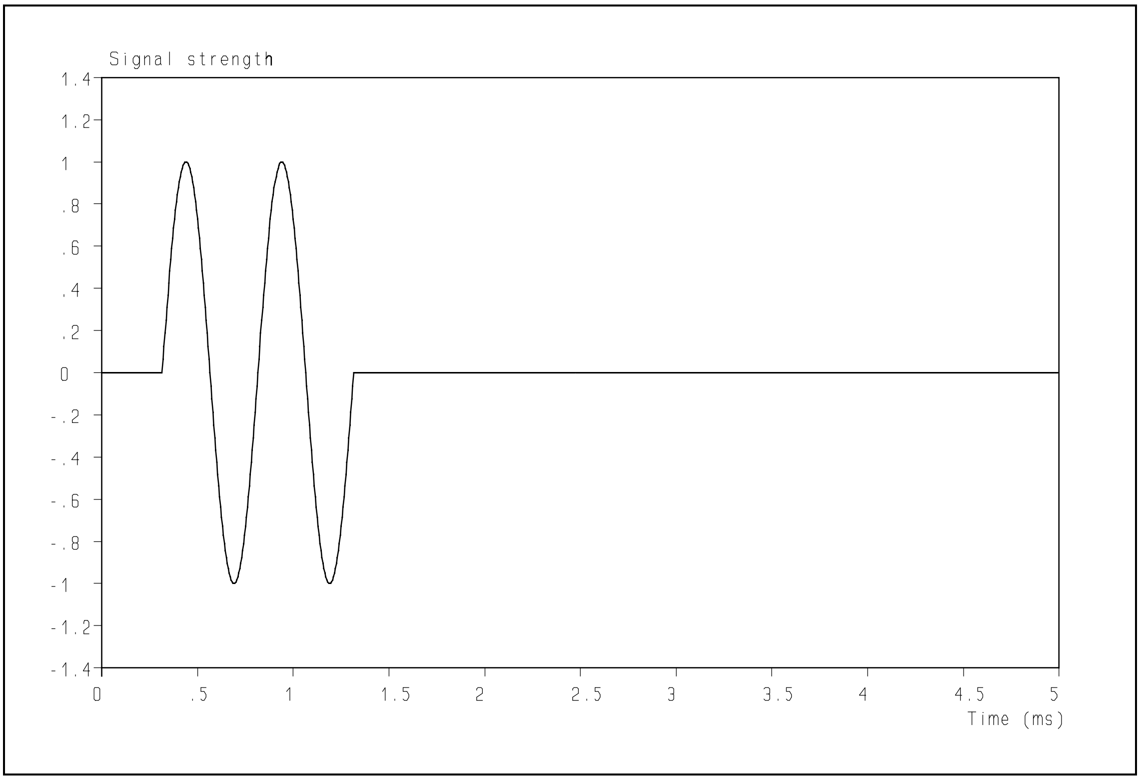

**Figure 2**: Input signal #1.

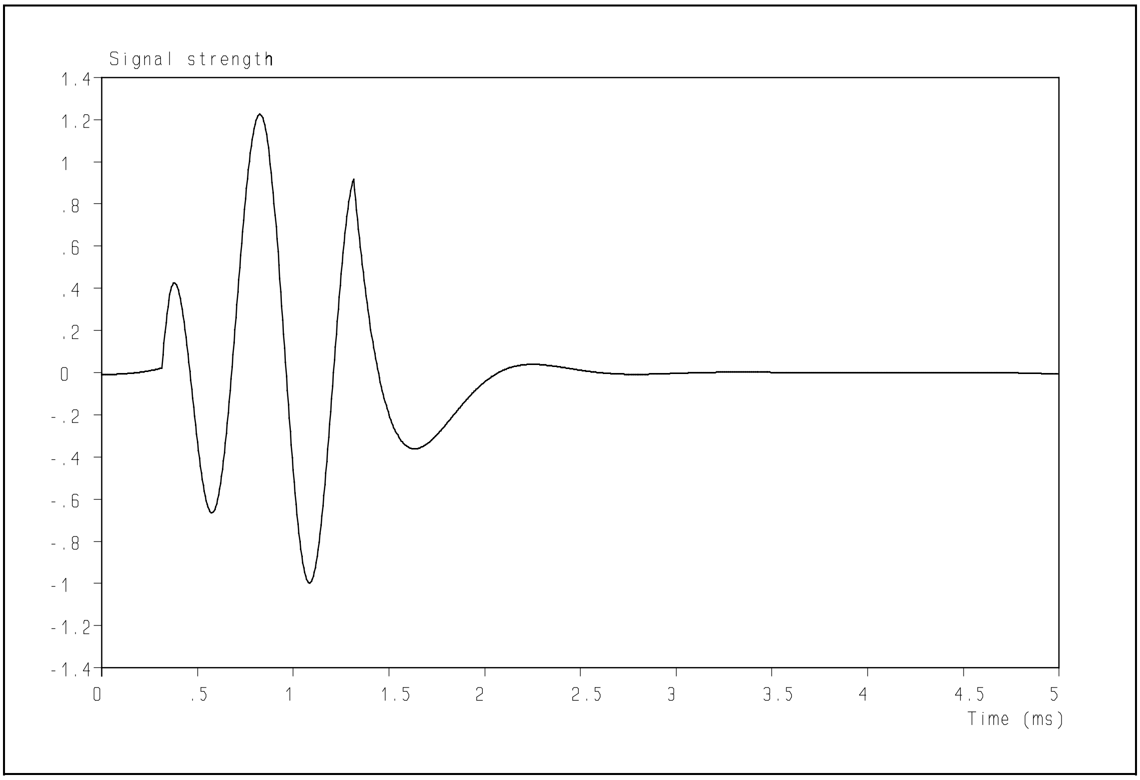


**Figure 3**: Input signal #2.

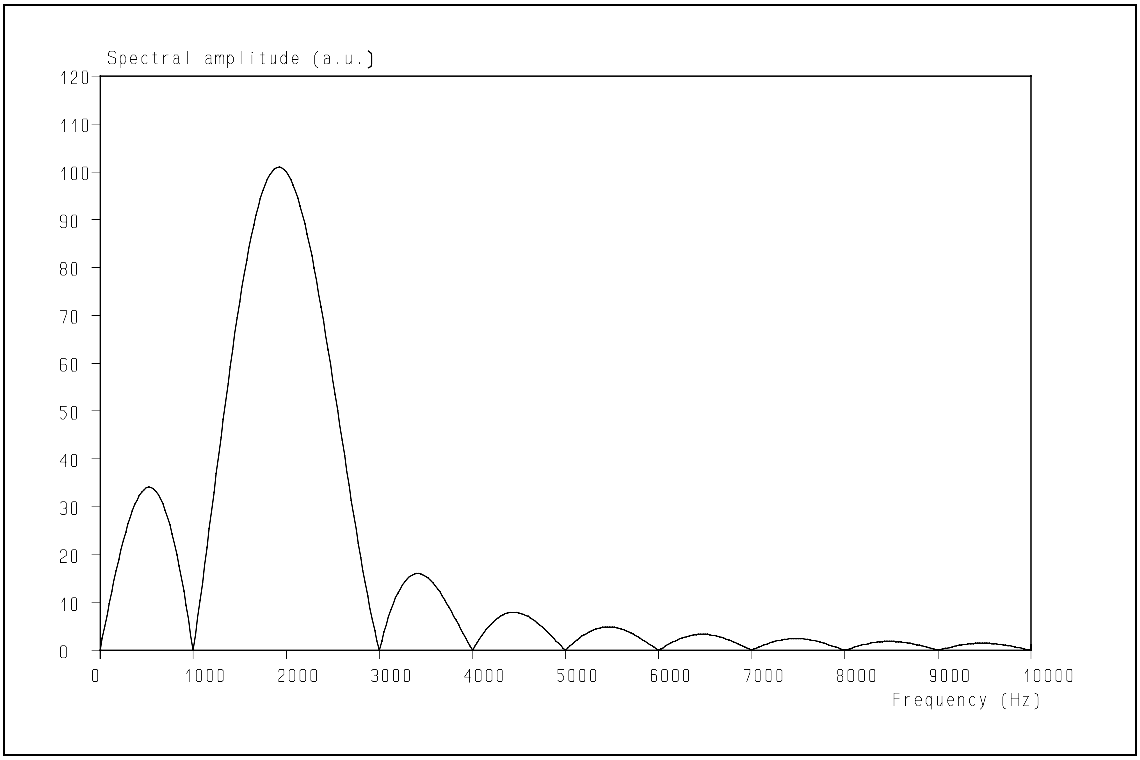


**Figure 4**: Spectrum of signal #1.

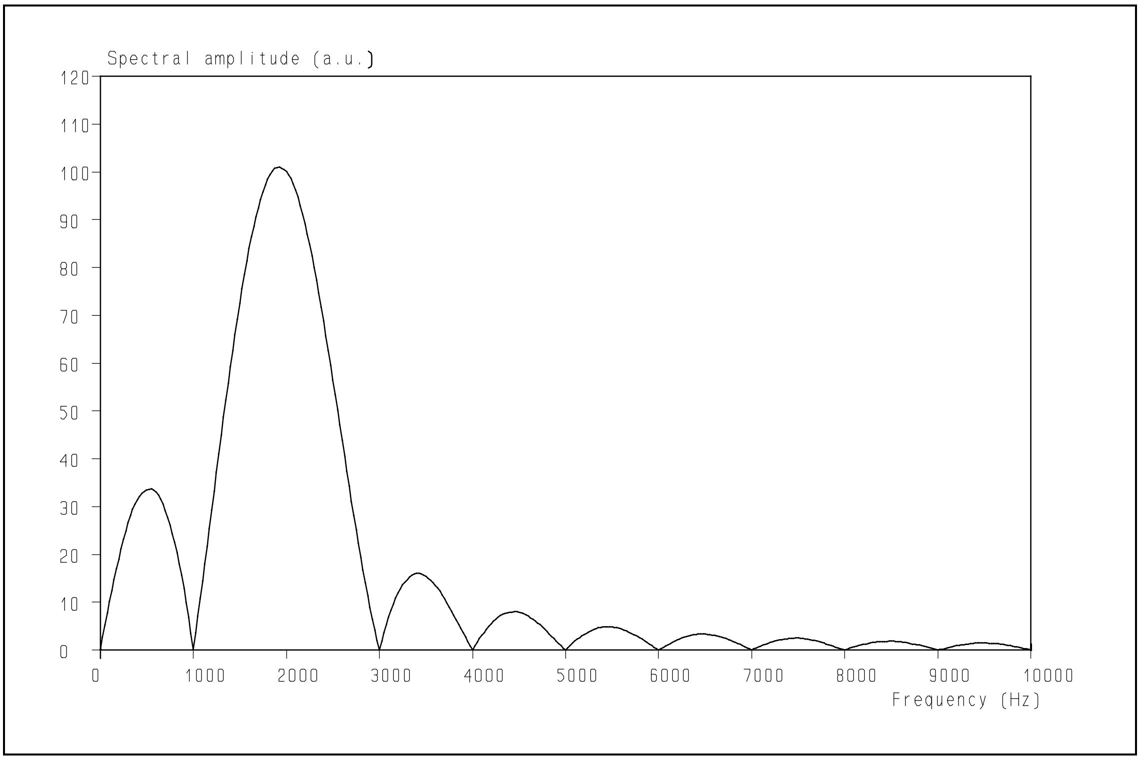


**Figure 5**: Spectrum of signal #2.

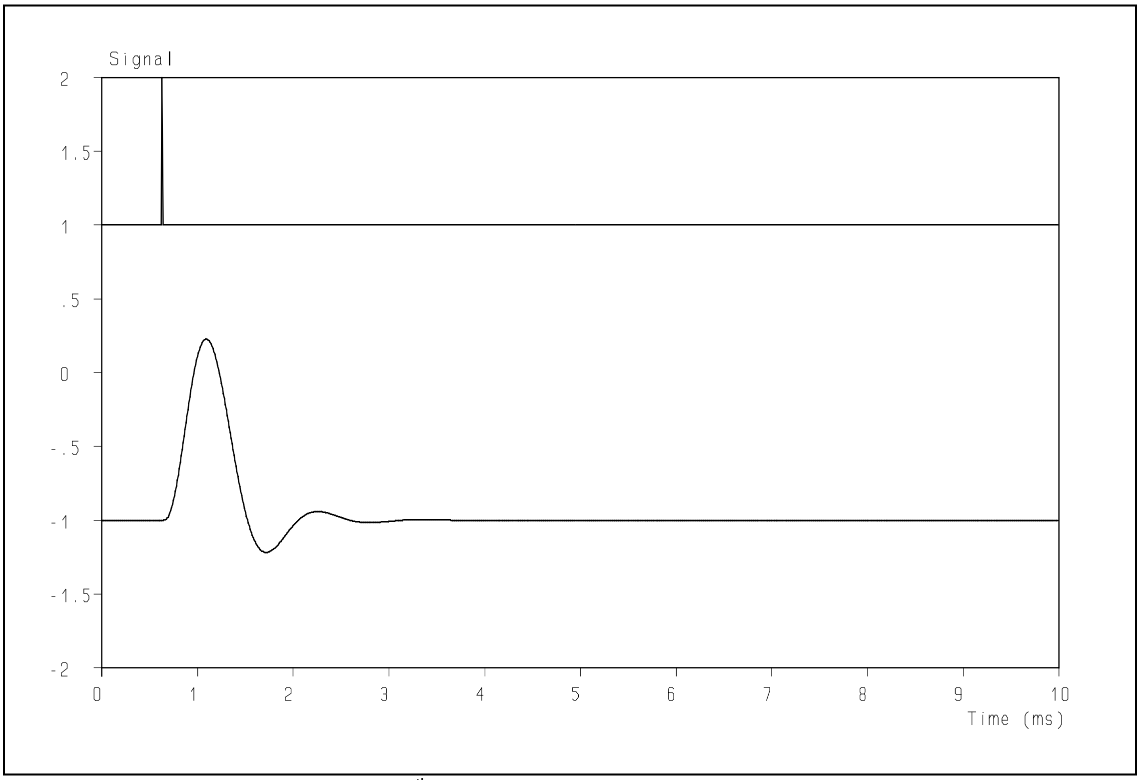


**Figure 6**: Impulse response of a $4^{th}$ order Butterworth filter.

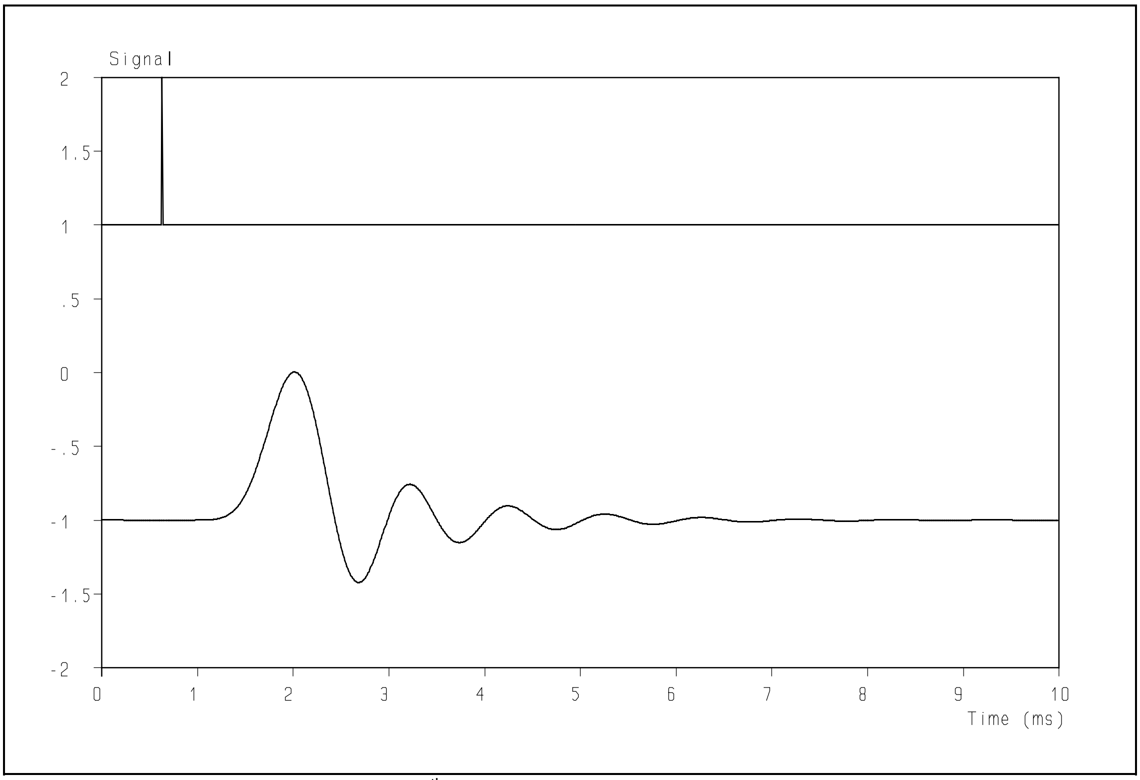


**Figure 7**: Impulse response of a 12th order Butterworth filter.

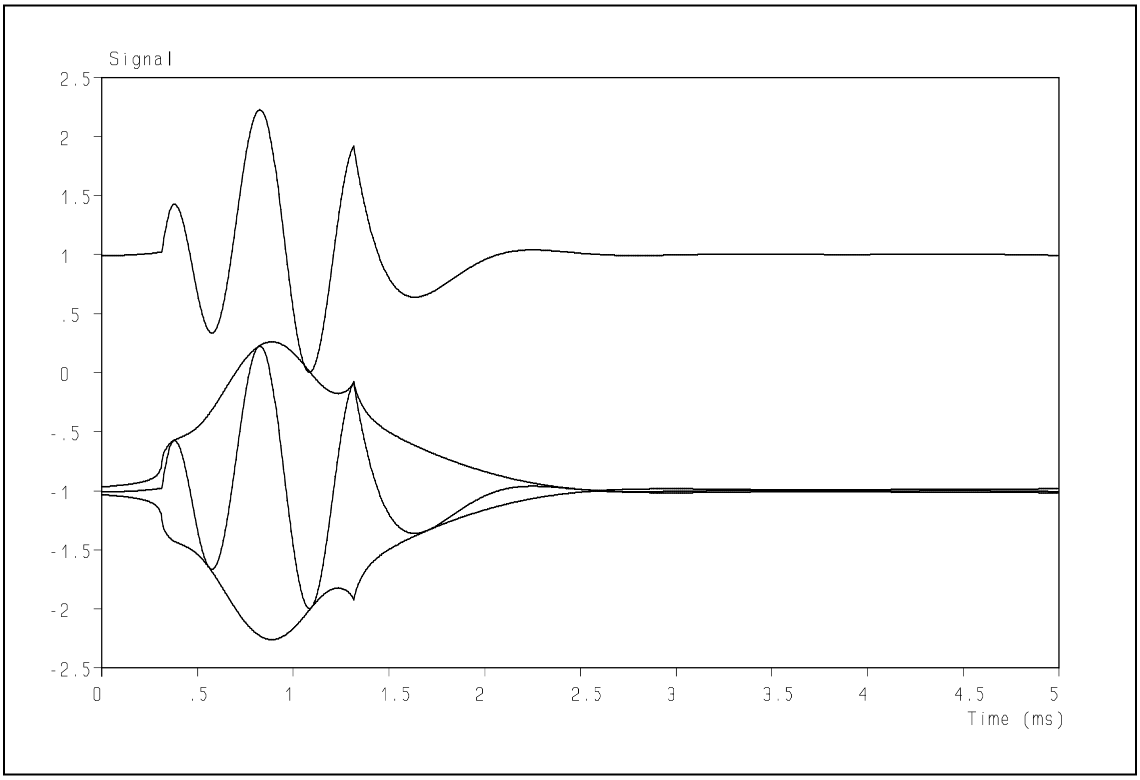


**Figure 8**: Signal #2 including its envelope.

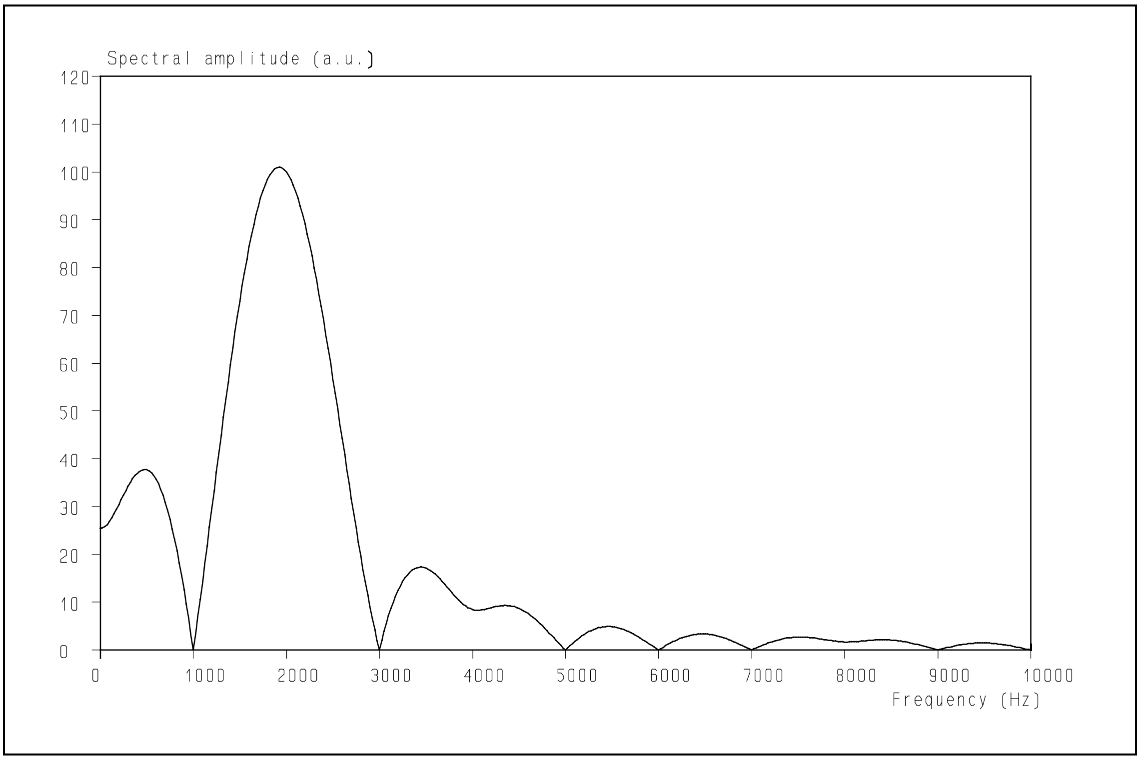


**Figure 9**: Spectrum of distorted signal #1.

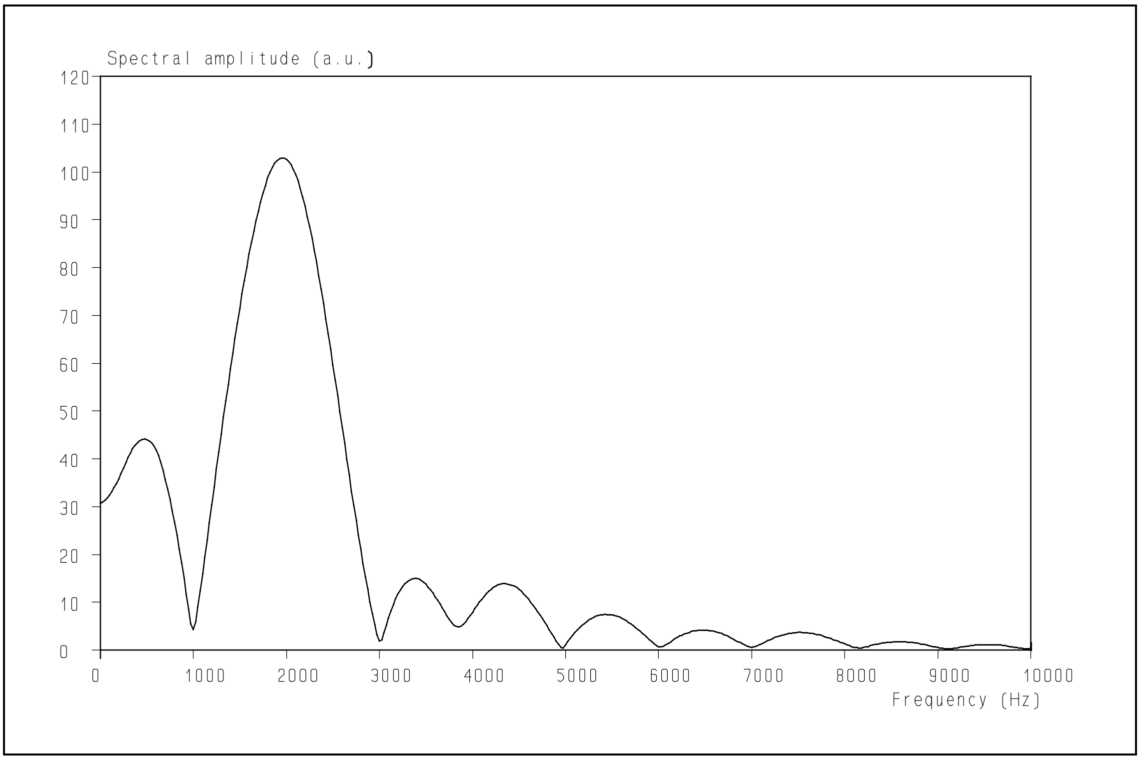


**Figure 10**: Spectrum of distorted signal #2.

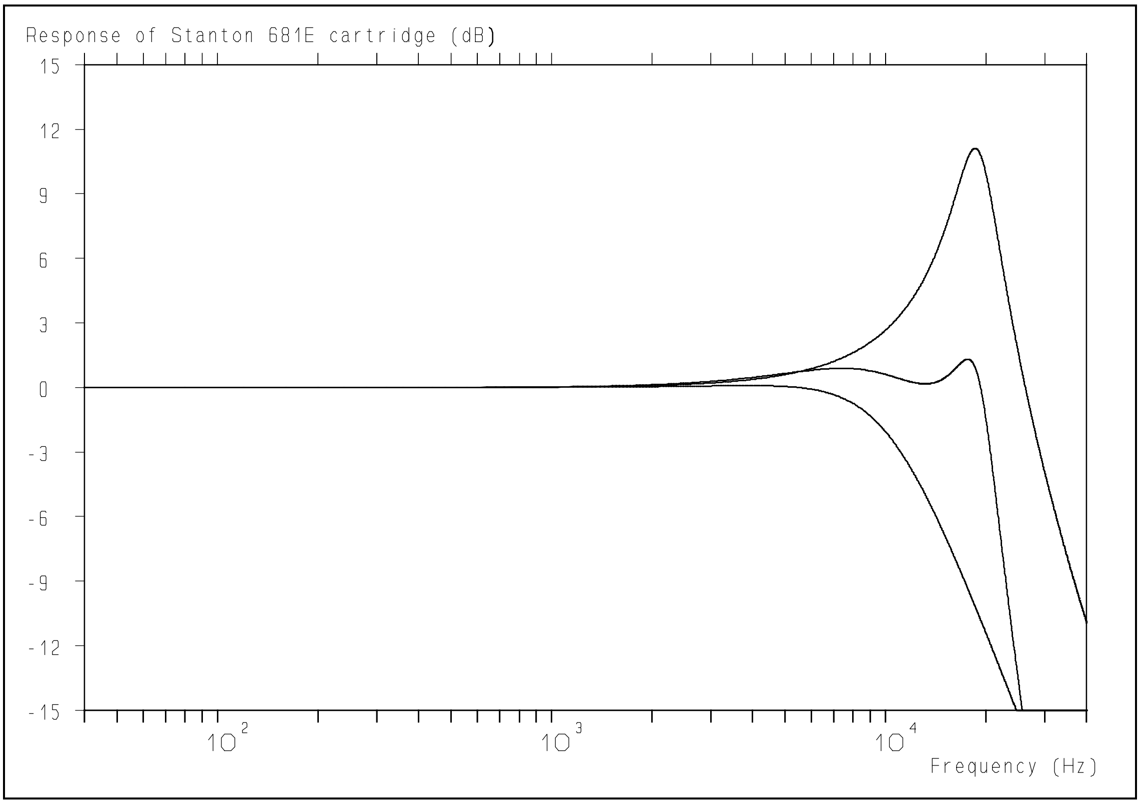


**Figure 11**: Response of Stanton 681E cartridge. Lower curve = electrical response, upper curve = mechanical response, middle curve = overall response.

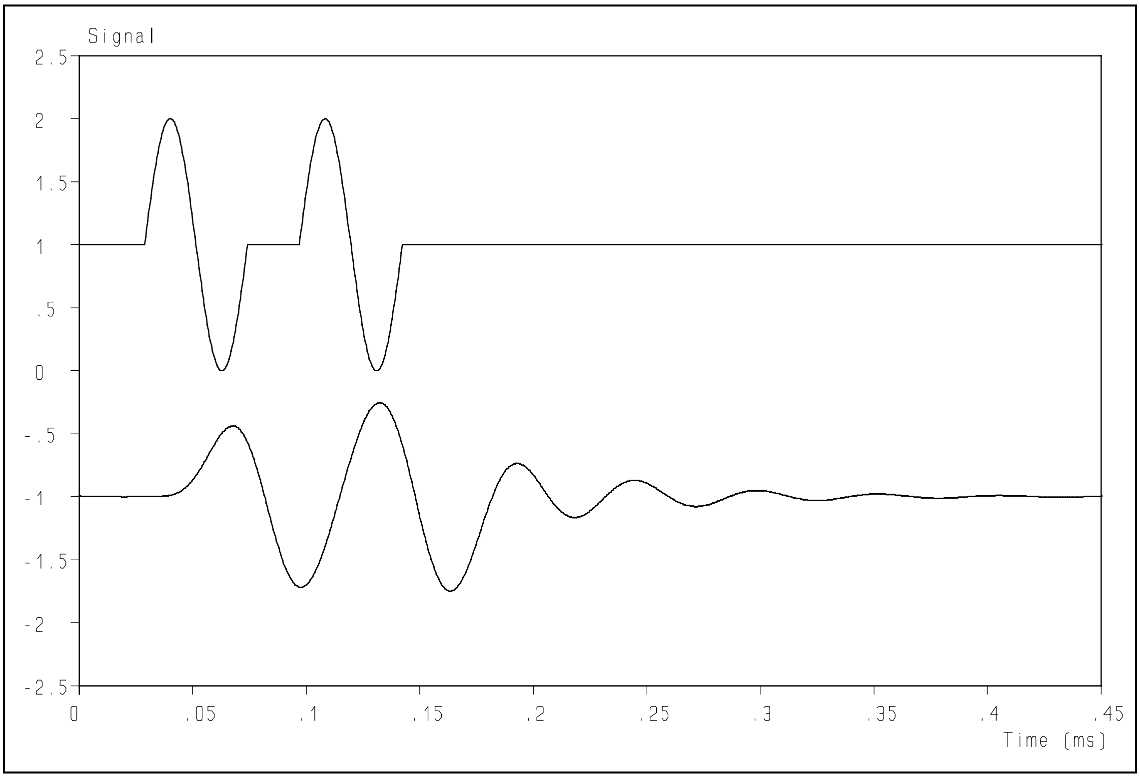


**Figure 12**: Response of a moving magnet cartridge to a complex signal.

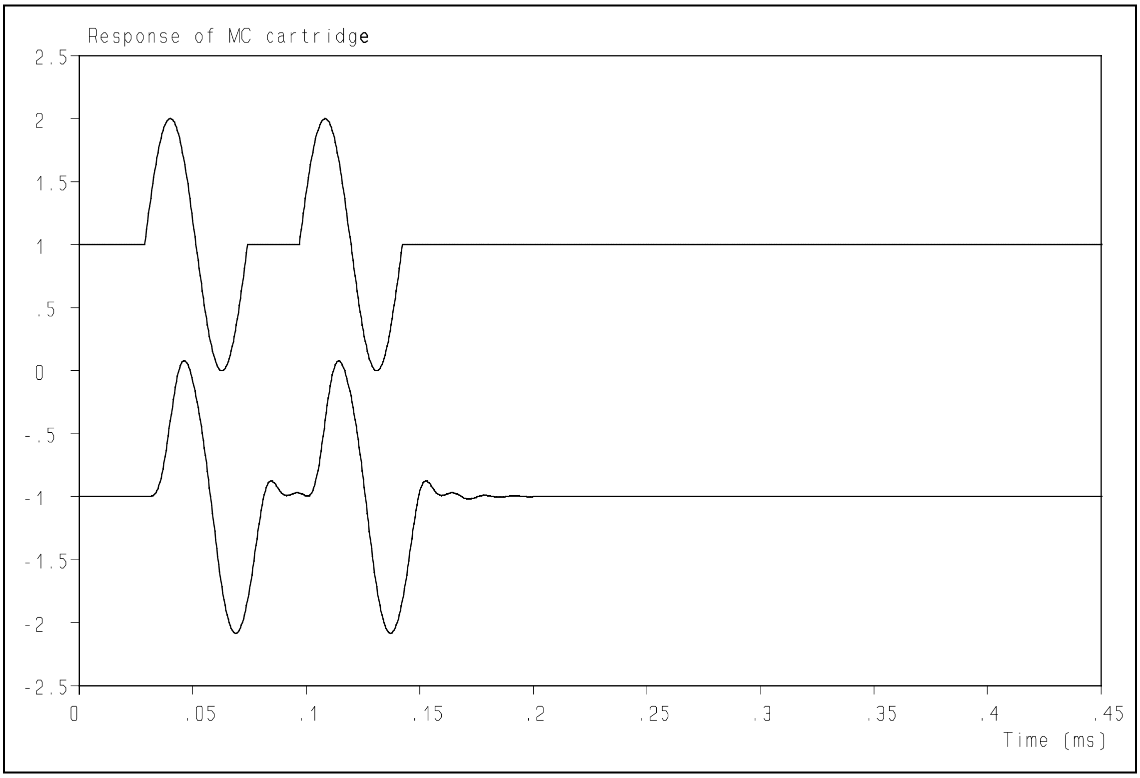

**Figure 13**: Response of a moving coil cartridge to a complex signal.

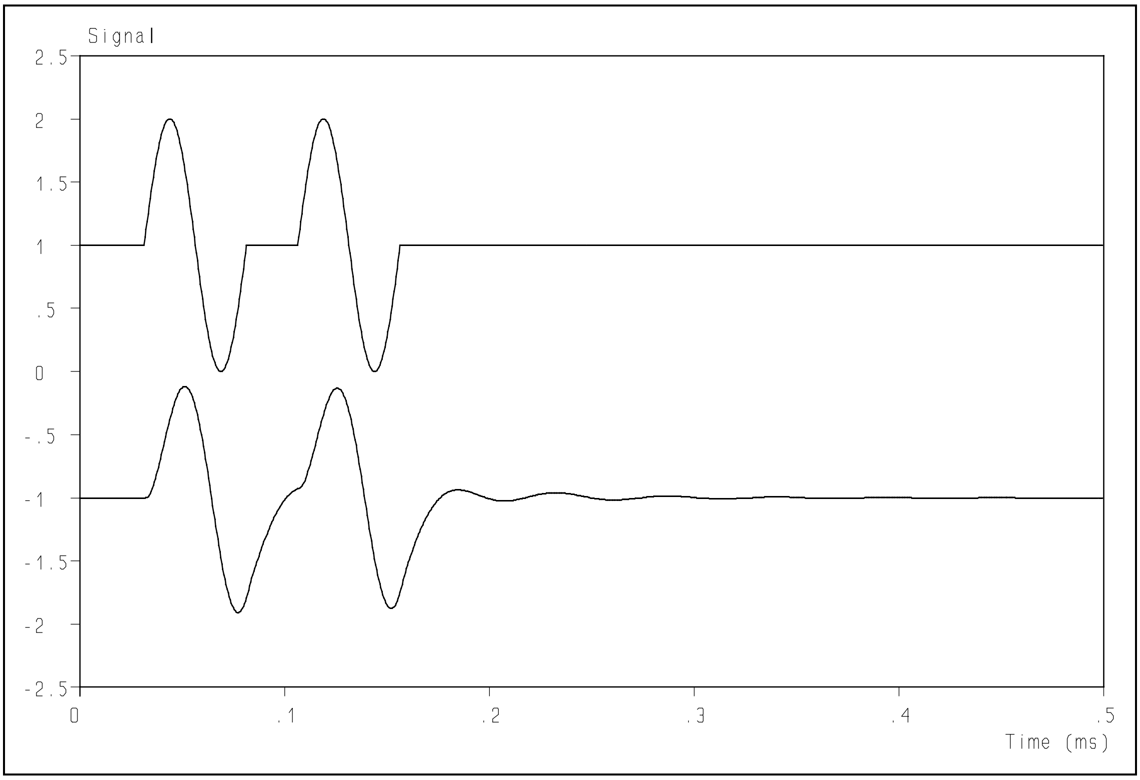

**Figure 14**: Response of a moving magnet cartridge to a complex signal after compensation.

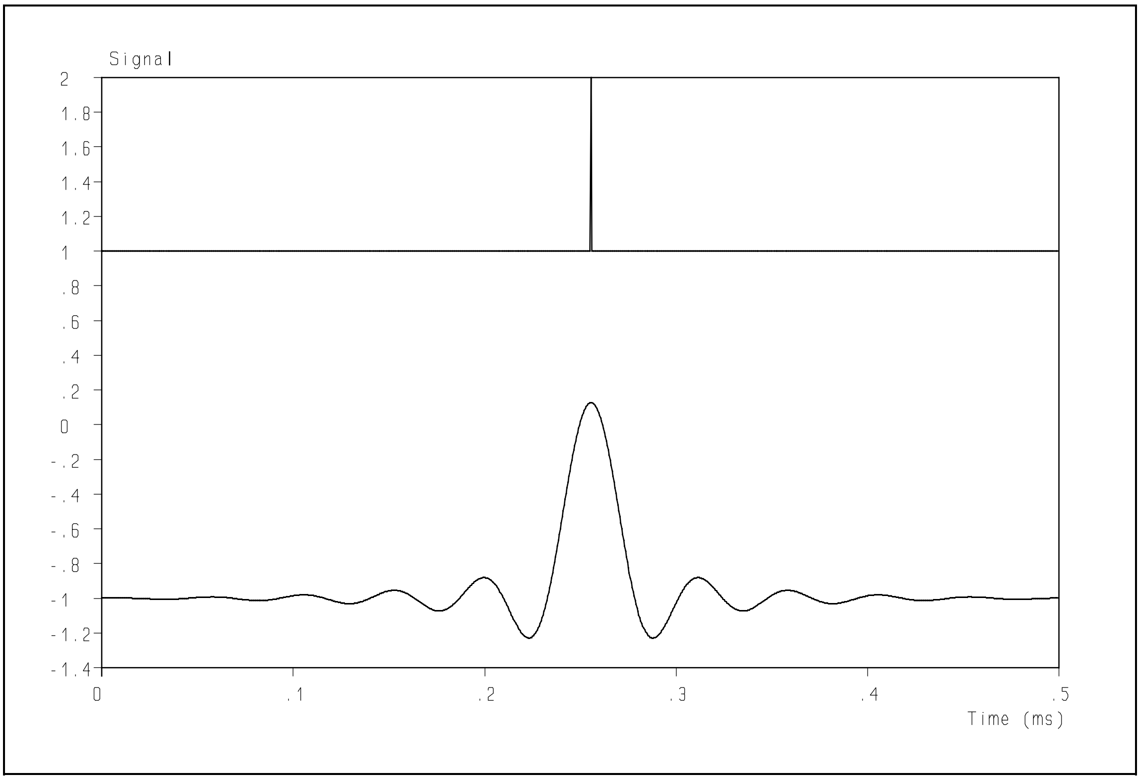


**Figure 15**: Impulse response of a CD reconstruction filter.

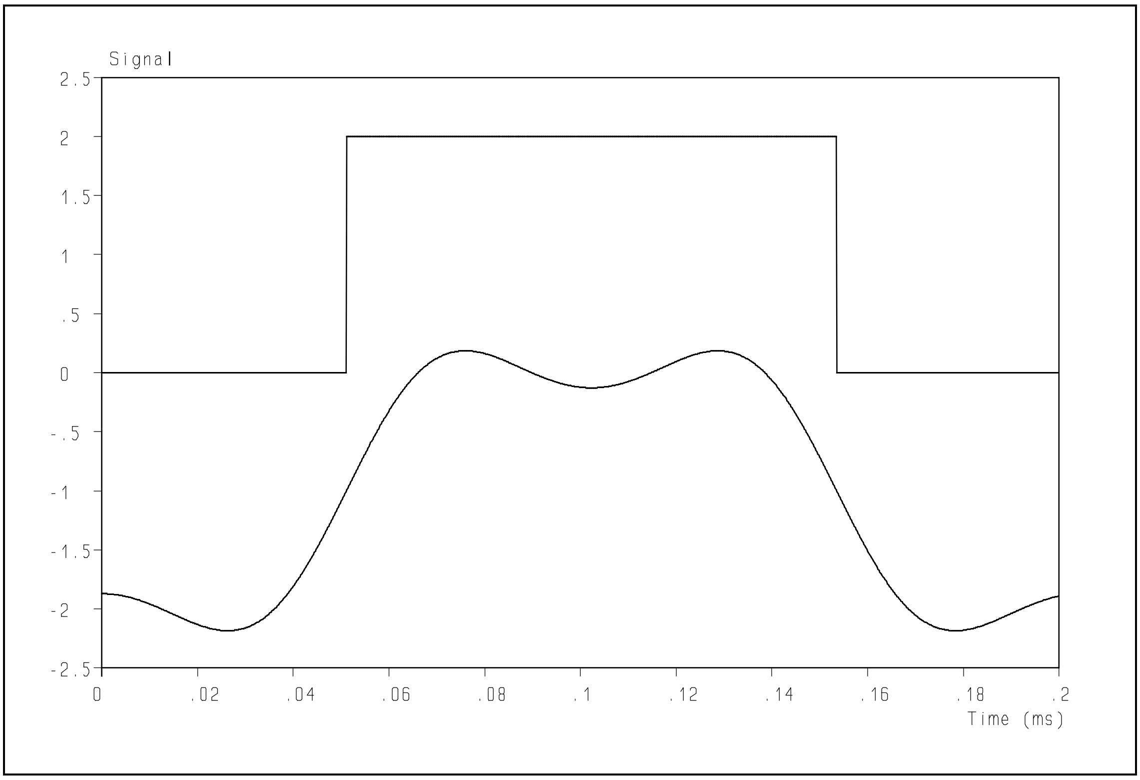


**Figure 16**: Step response of a CD reconstruction filter.

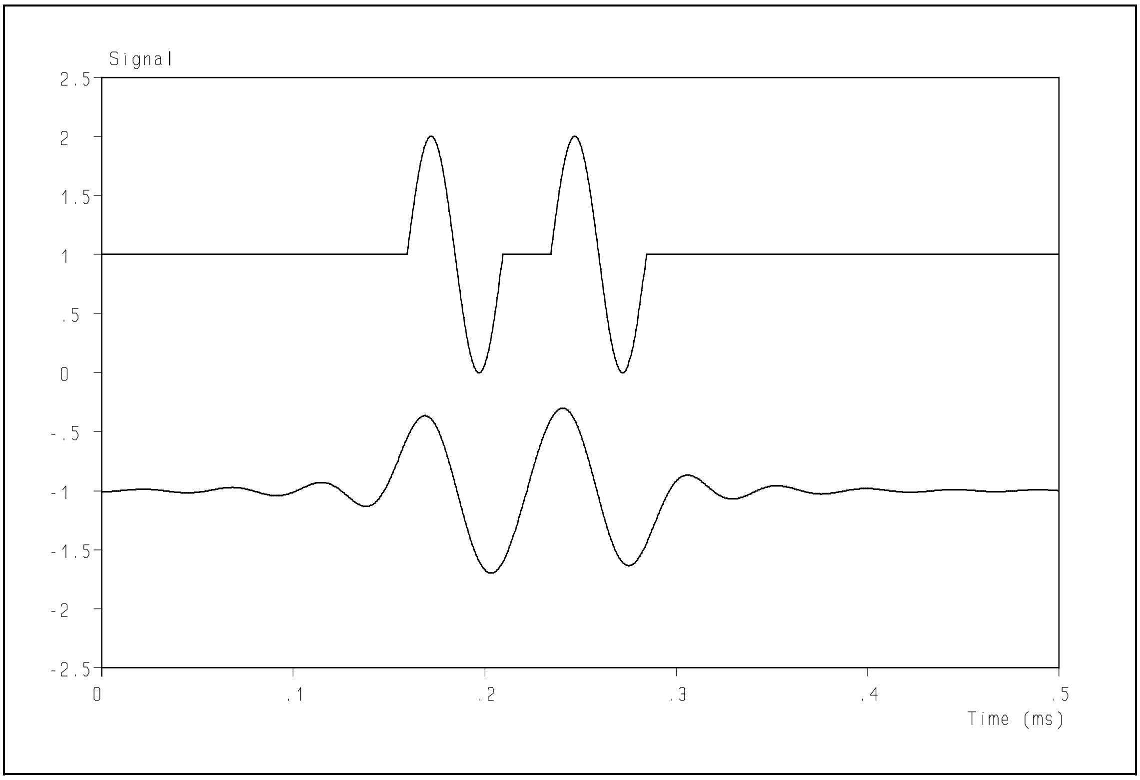

**Figure 17**: Response of a CD reconstruction filter to a complex signal.

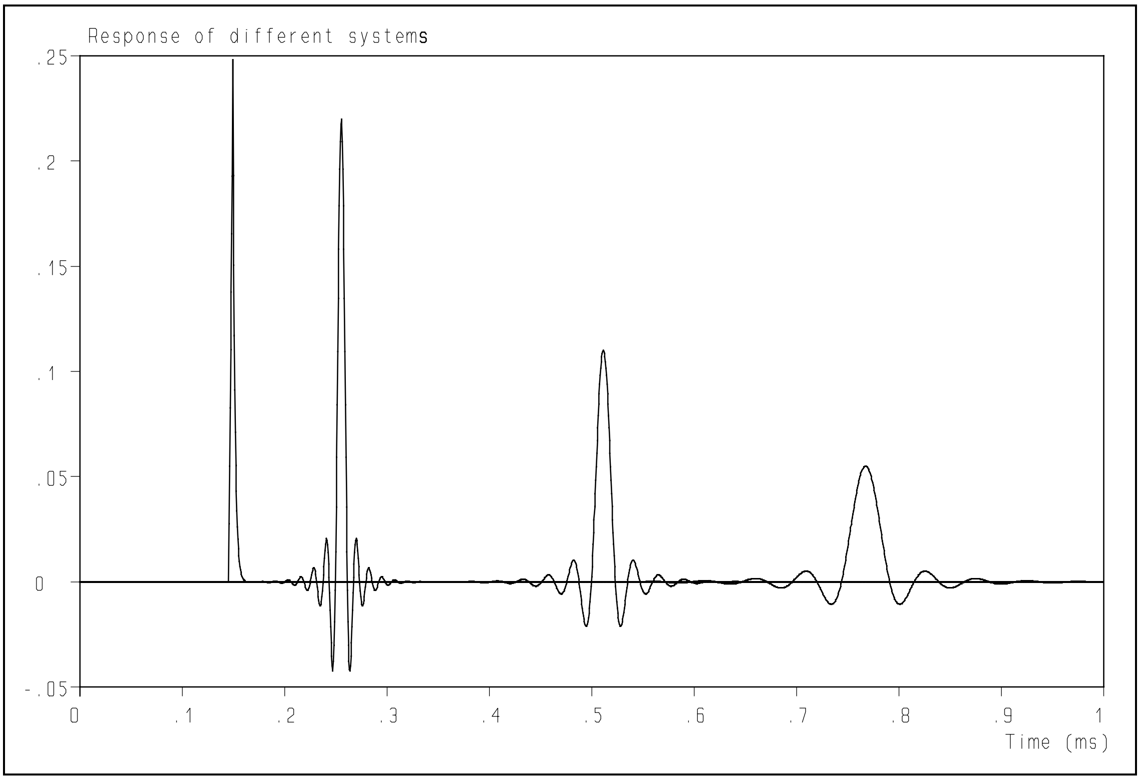

**Figure 18**: Responses (from left to right) of a DSD, a 192 kHz, a 96 kHz and a 48 kHz system on a -6 dB block input of 3 μs duration and an amplitude of 0.25 (from ref. 7).

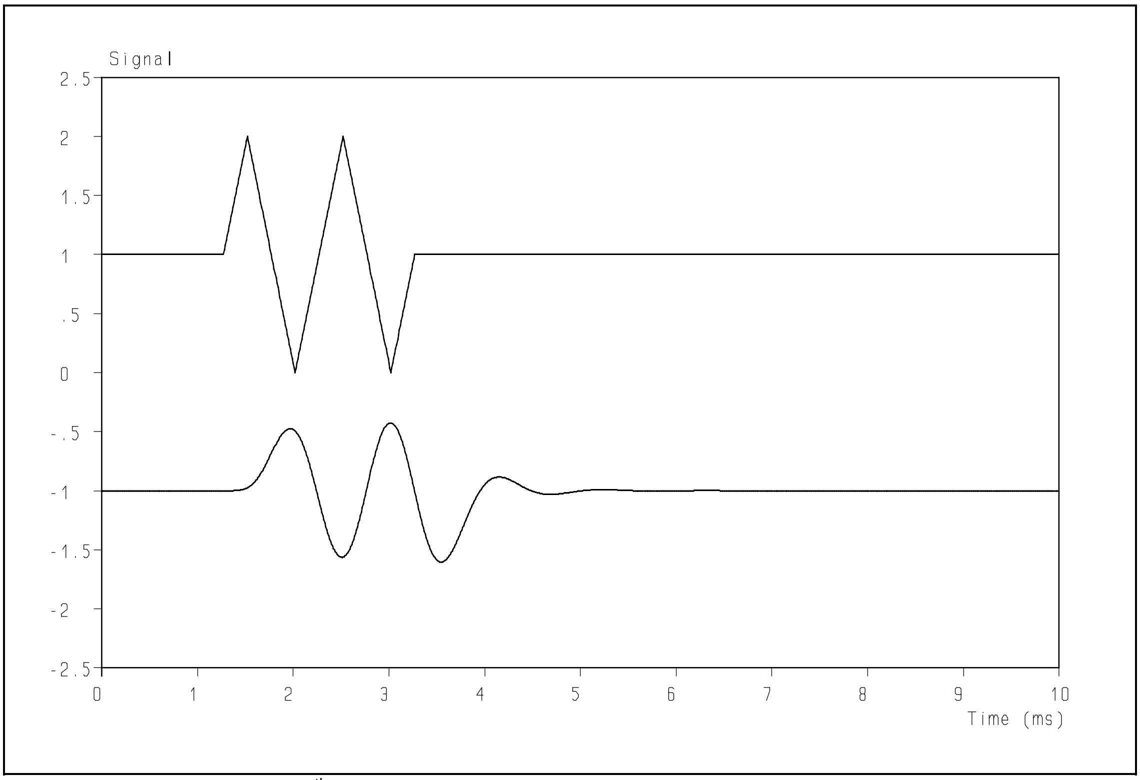

**Figure 19**: Response of 4th order Butterworth low-pass filter to a complex signal.

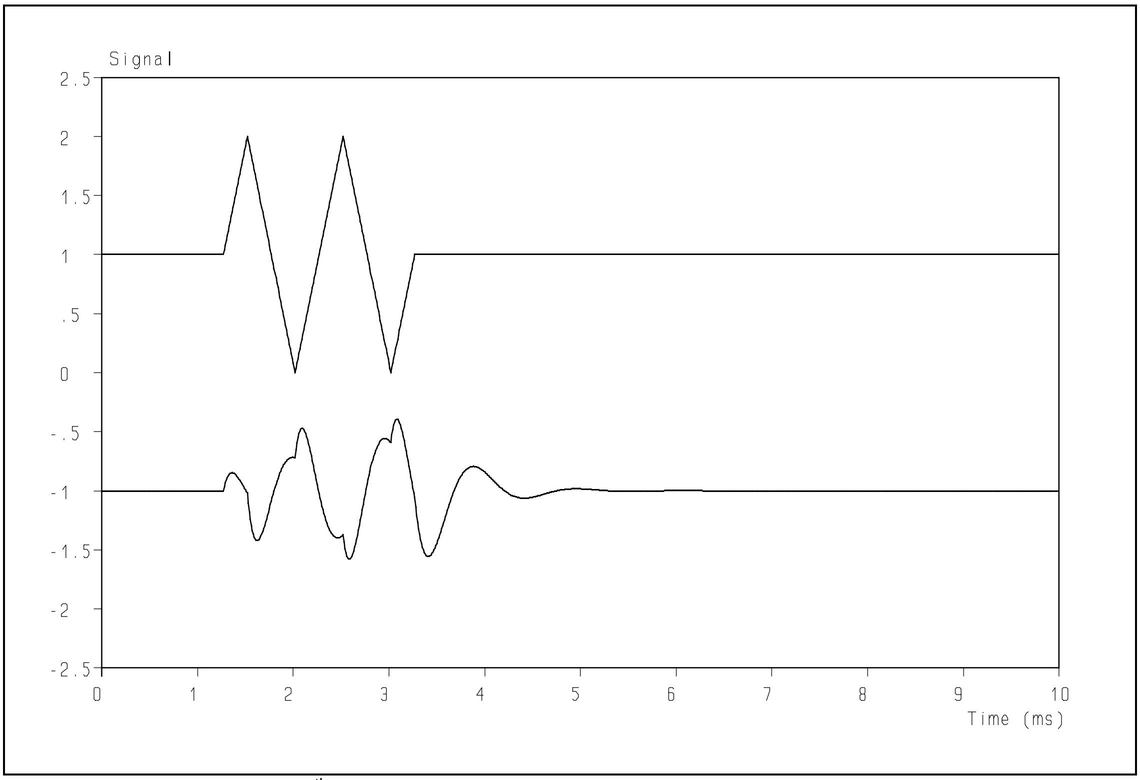

**Figure 20**: Response of 4th order Butterworth high-pass filter to a complex signal.

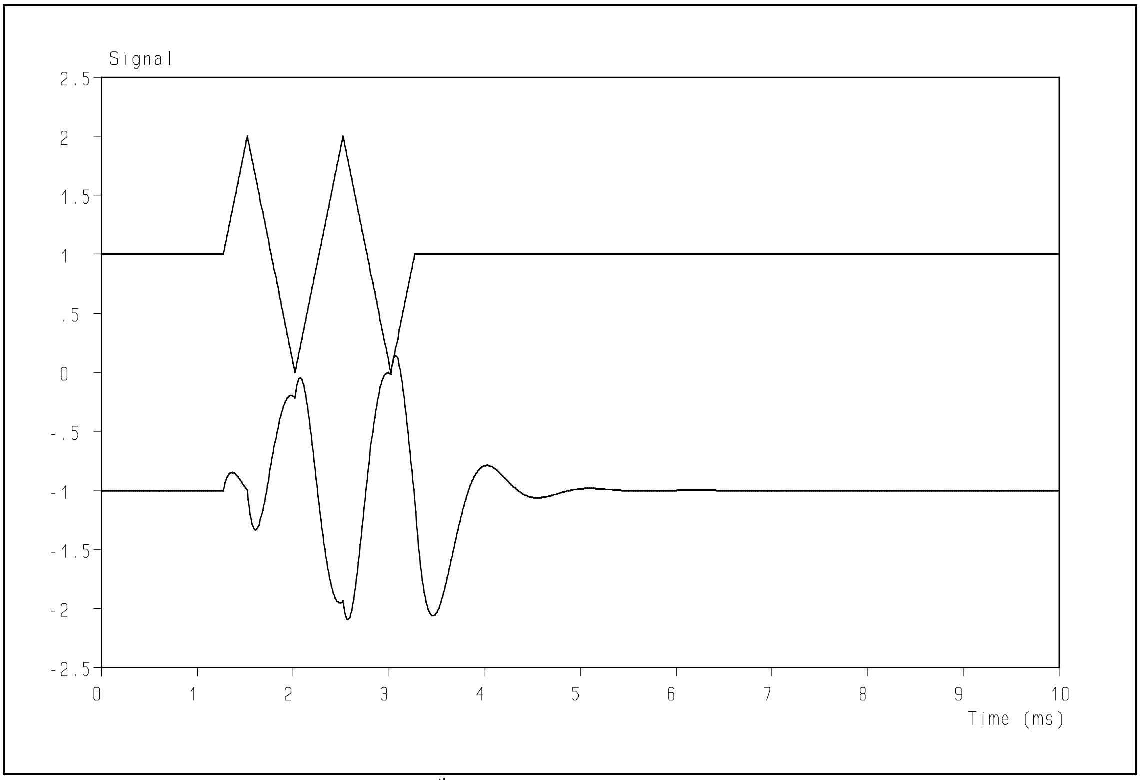


**Figure 21**: Combined response of 4th order Butterworth low and high-pass filters to a complex signal.

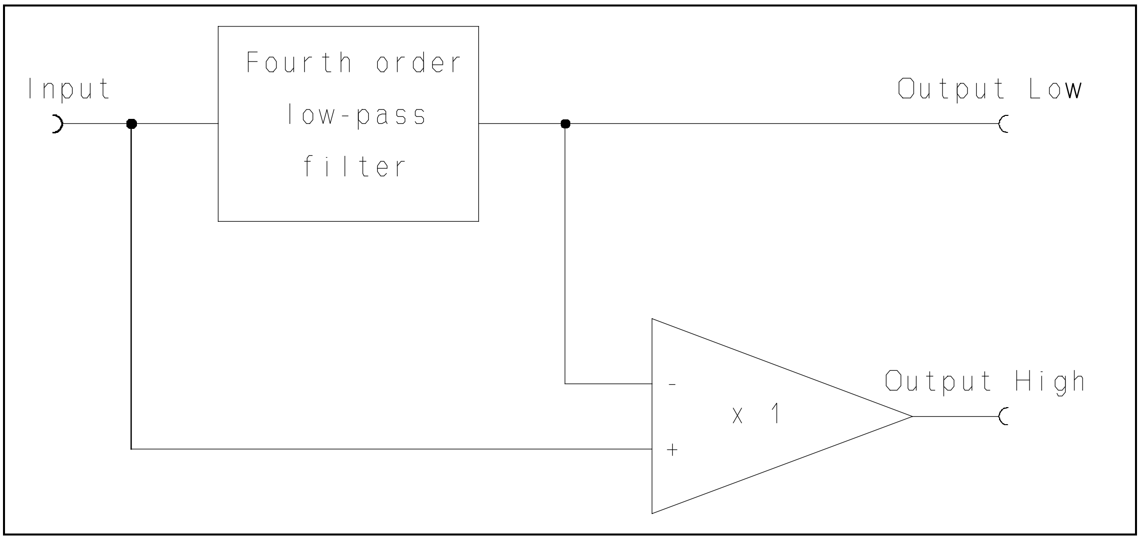


**Figure 22**: Schematic diagram of subtraction filter.

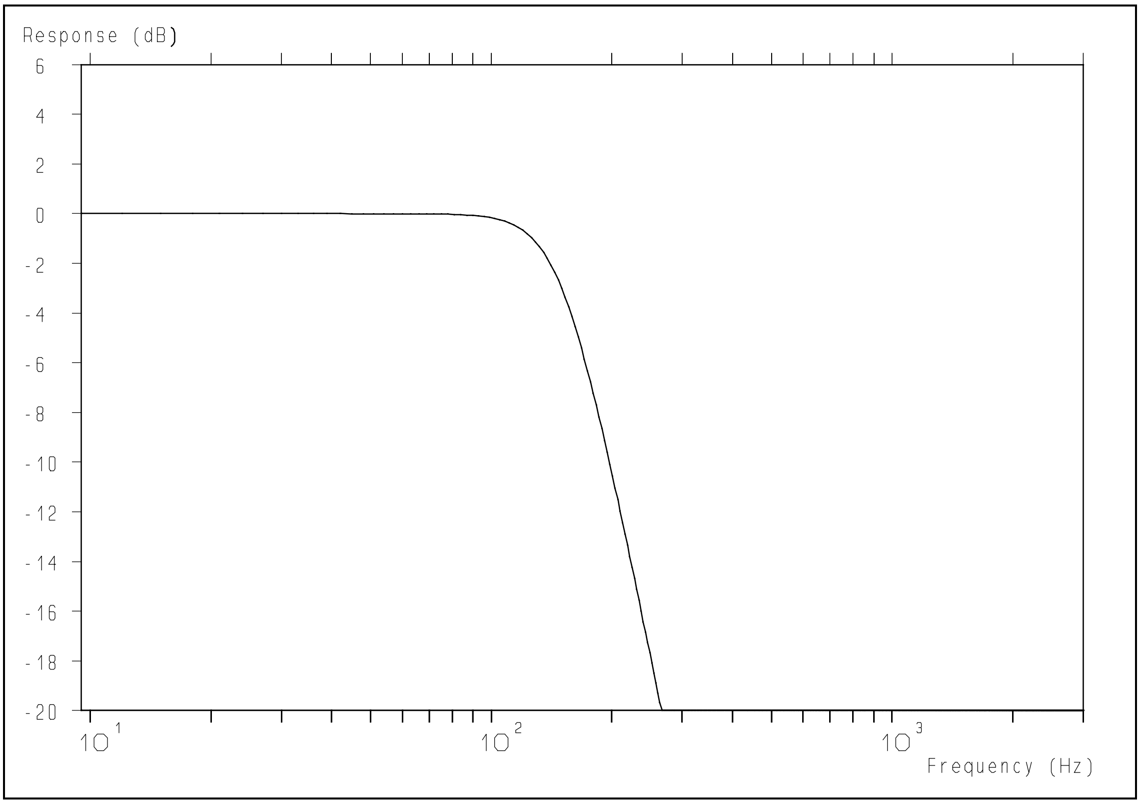


**Figure 23**: Response of fourth order Butterworth low-pass filter of 150 Hz.

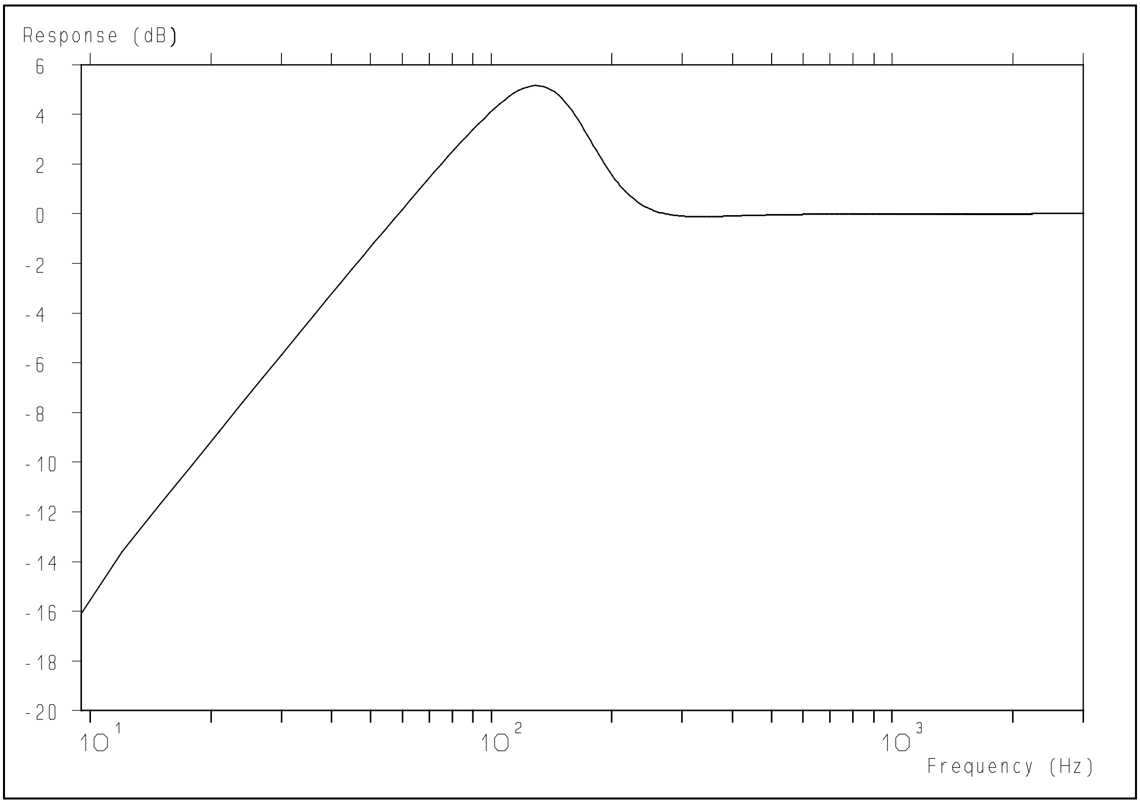


**Figure 24**: Response of the complement of the fourth order Butterworth low-pass filter of 150 Hz.

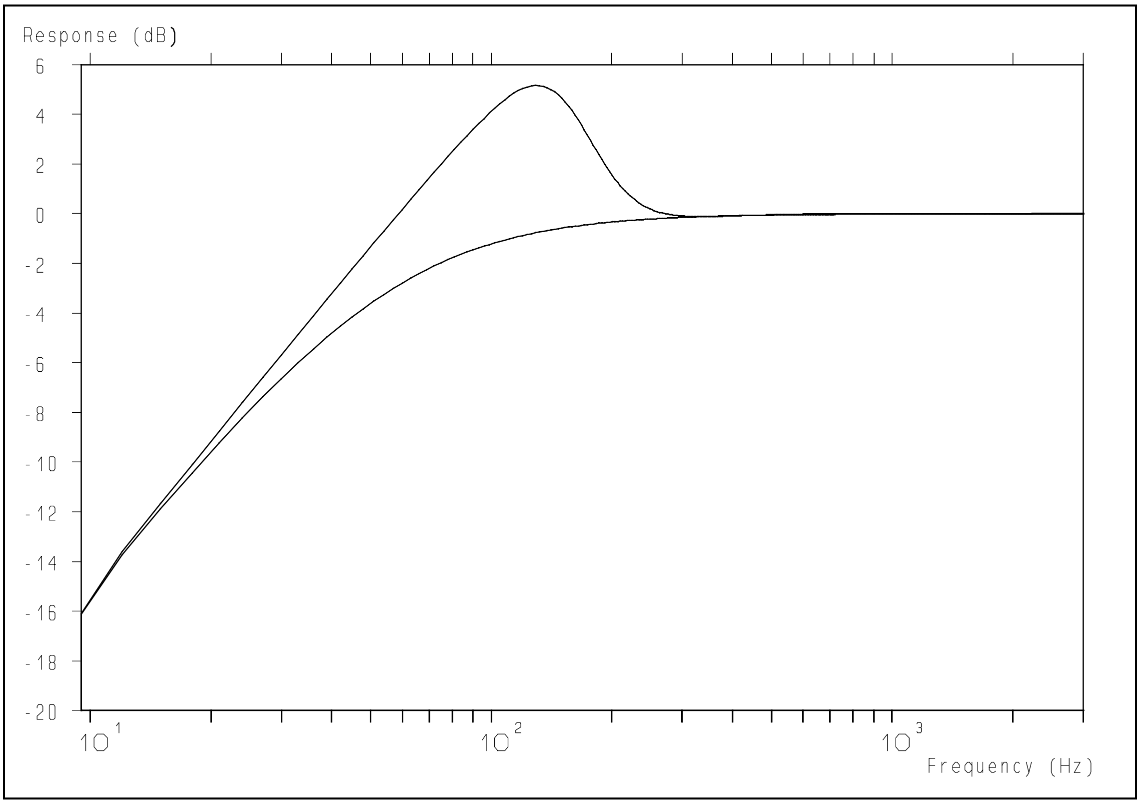


**Figure 25**: Response of complement of fourth order Butterworth low-pass filter of 150 Hz and a first-order high pass filter of 57 Hz.

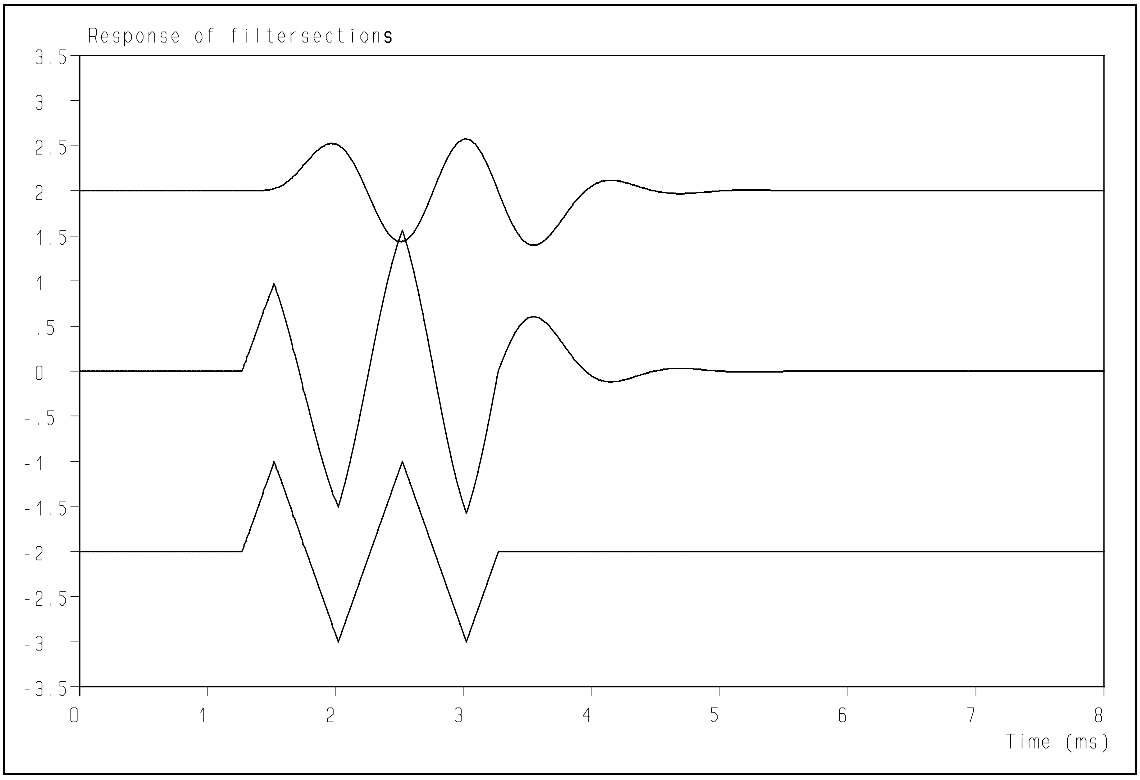


**Figure 26**: Response of fourth order Butterworth low-pass filter (upper trace), its complement (centre) and the summed response (lower trace).

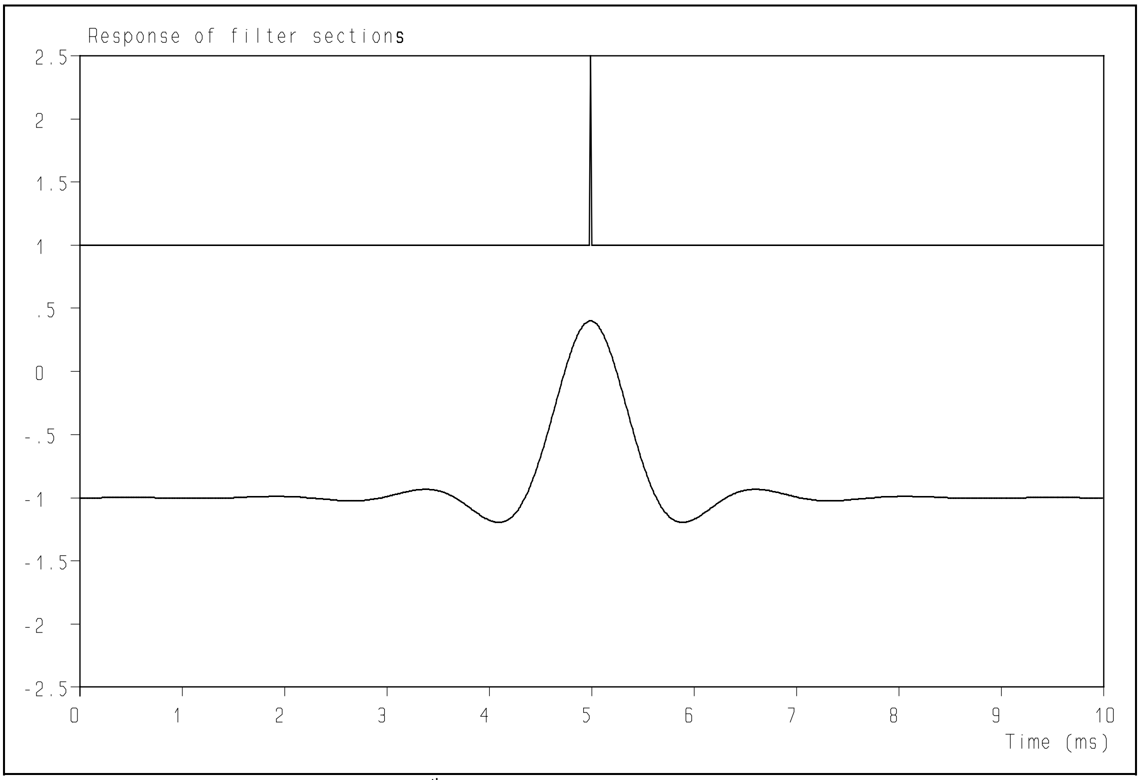

**Figure 27**: Impulse response of a 6th order digital Butterworth low-pass filter.

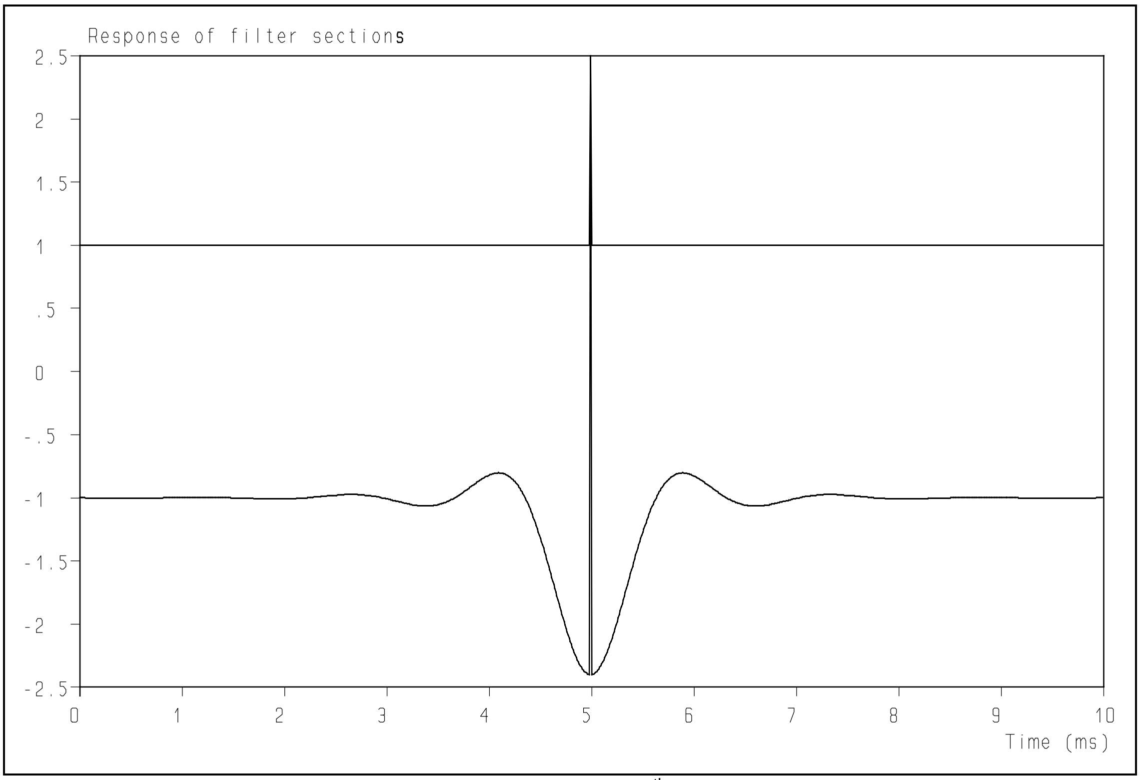

**Figure 28**: Impulse response of the complement of a 6th order digital Butterworth low-pass filter. (N.B. The peak a5 5 ms. is present in both upper and lower trace)

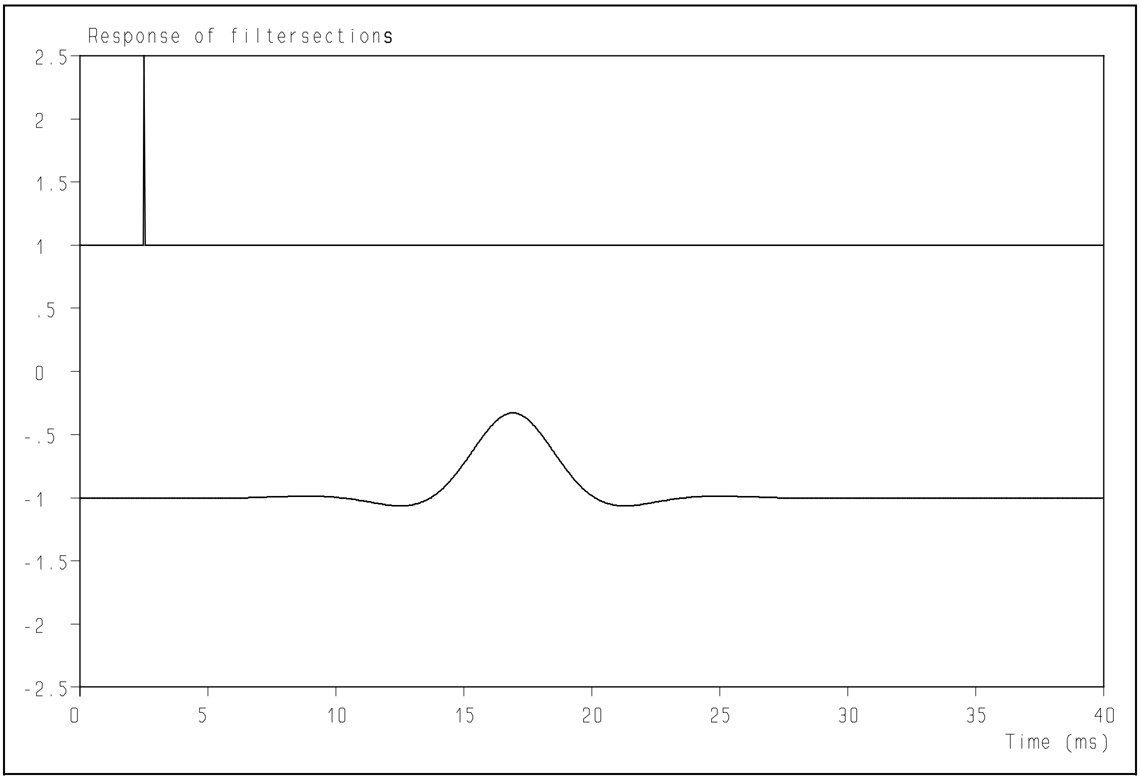


**Figure 29**: Example of phase-linear causal low-pass filter with "pre"-ringing.

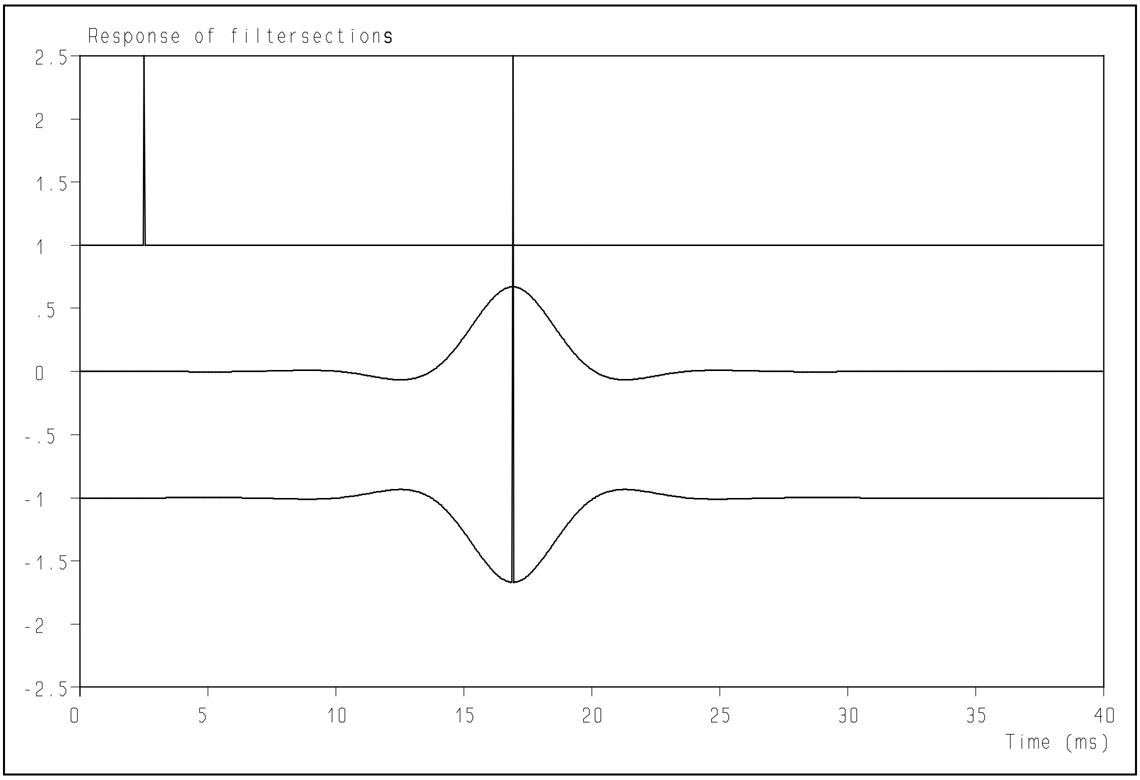


**Figure 30**: Example of phase-linear causal filters with "pre"-ringing which require the two loudspeaker units to create complete silence before the onset of the sound. (N.B. The peak at 17 ms. is part of the lowest trace)

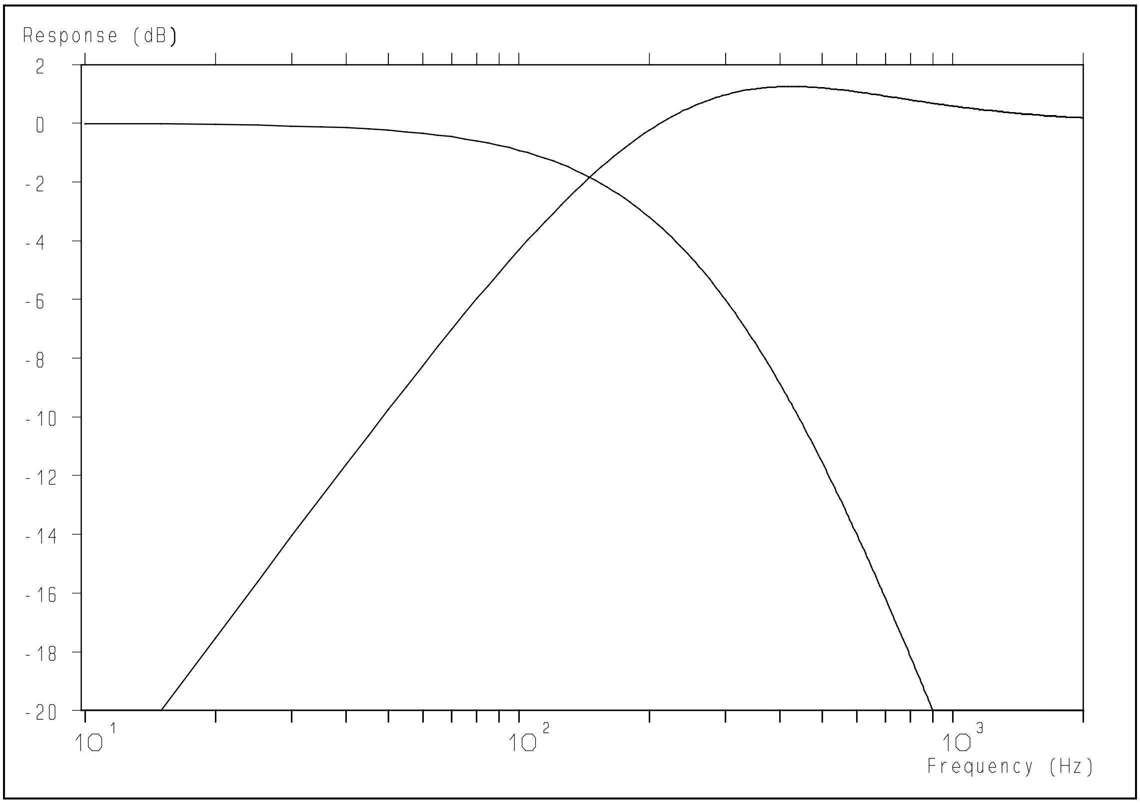


**Figure 31**: Response of a second-order analog low-pass filter and its complement to create a time-correct overall response.

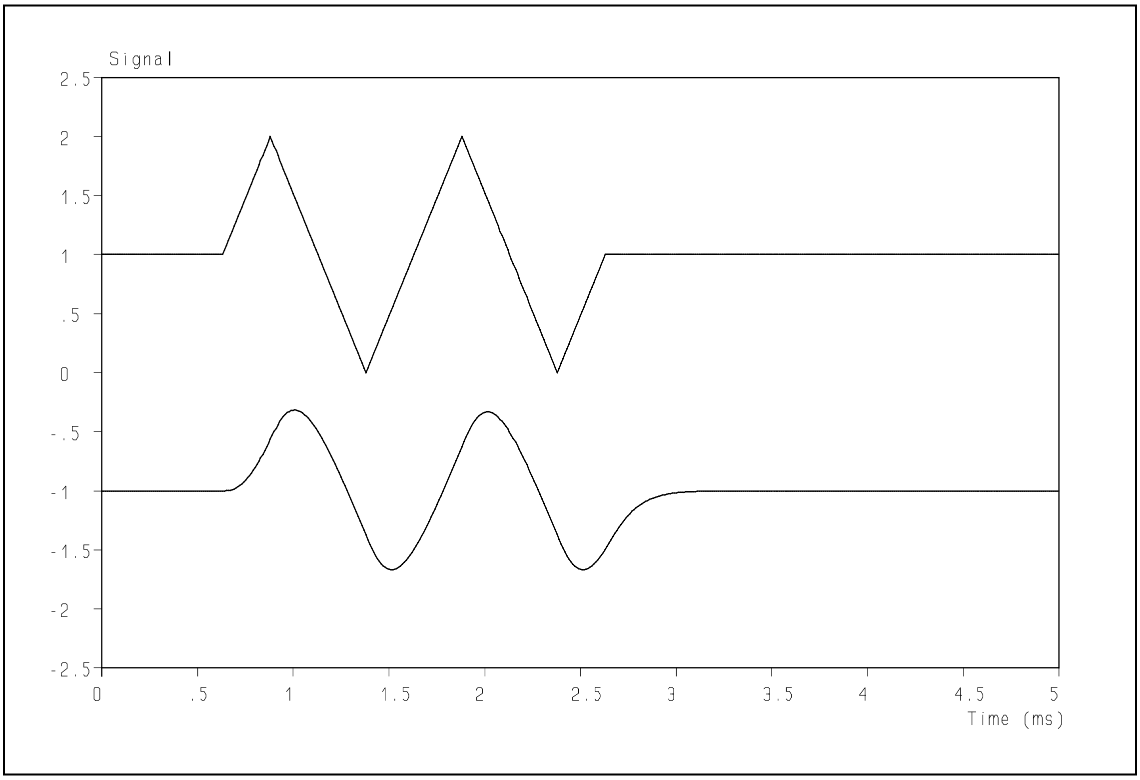


**Figure 32**: Response of low-pass section of phase-correct analog filter (second order).

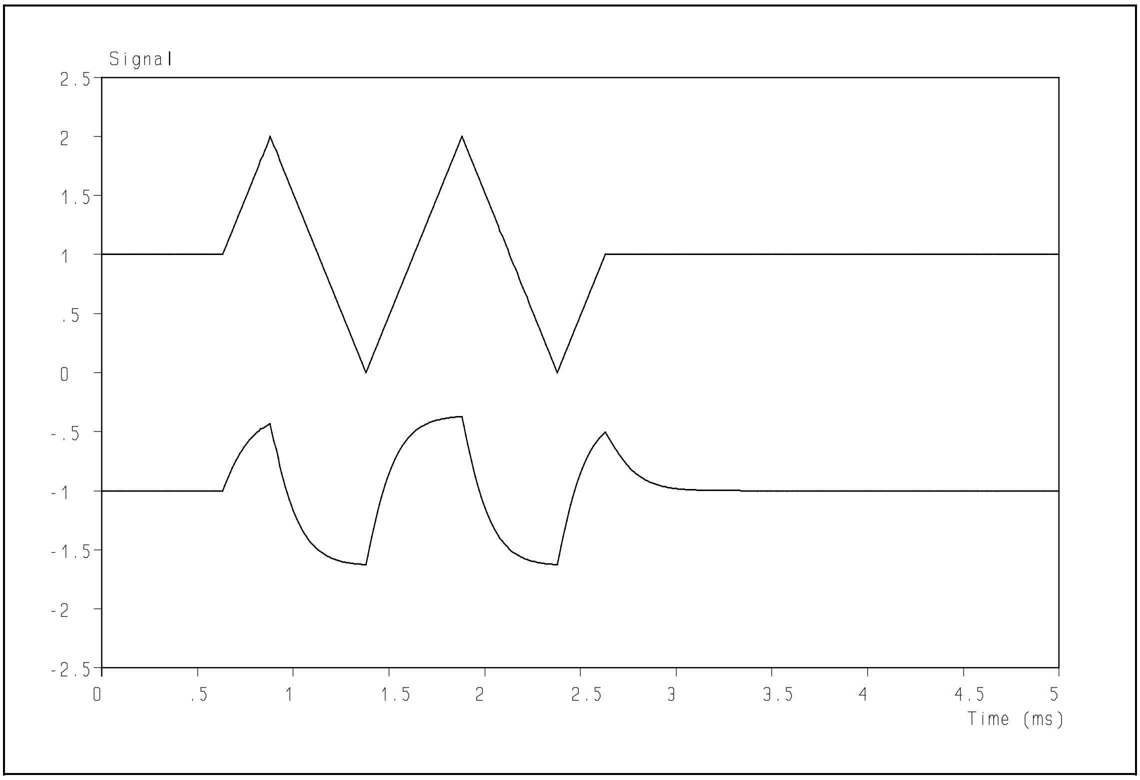

**Figure 33**: Response of high-pass section of phase-correct analog filter (less then first order).

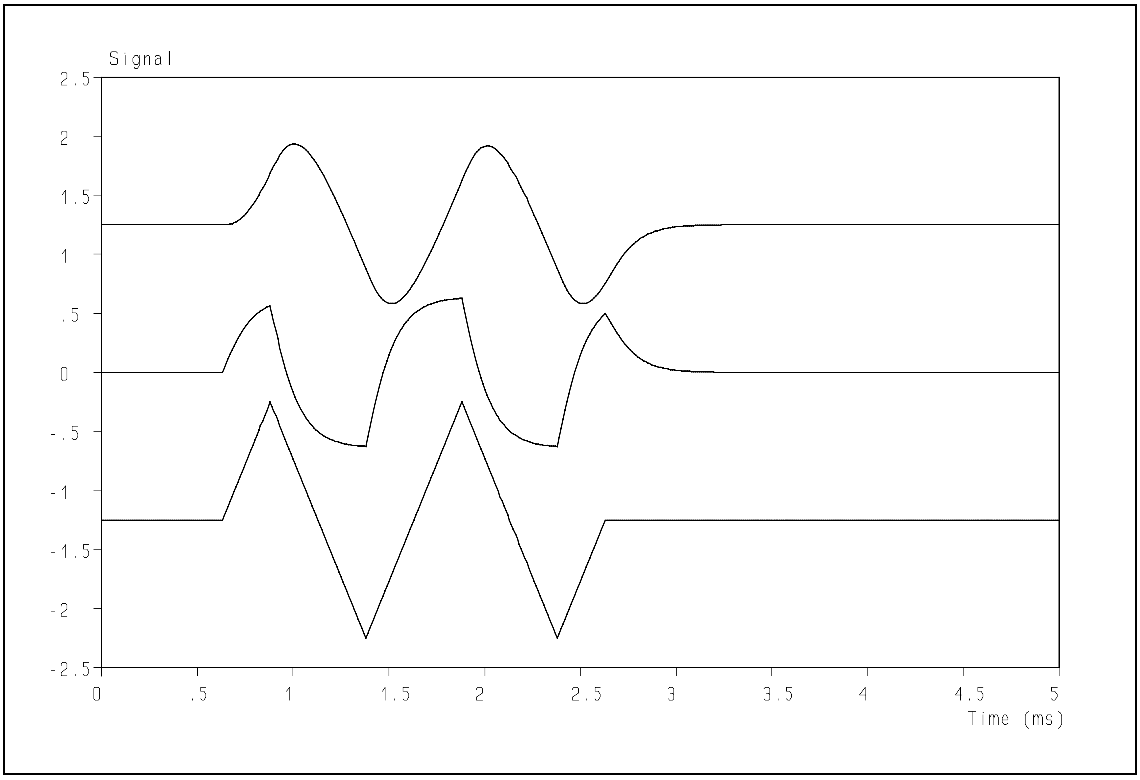

**Figure 34**: Combined response of low and high-pass sections of phase-correct analog filter.

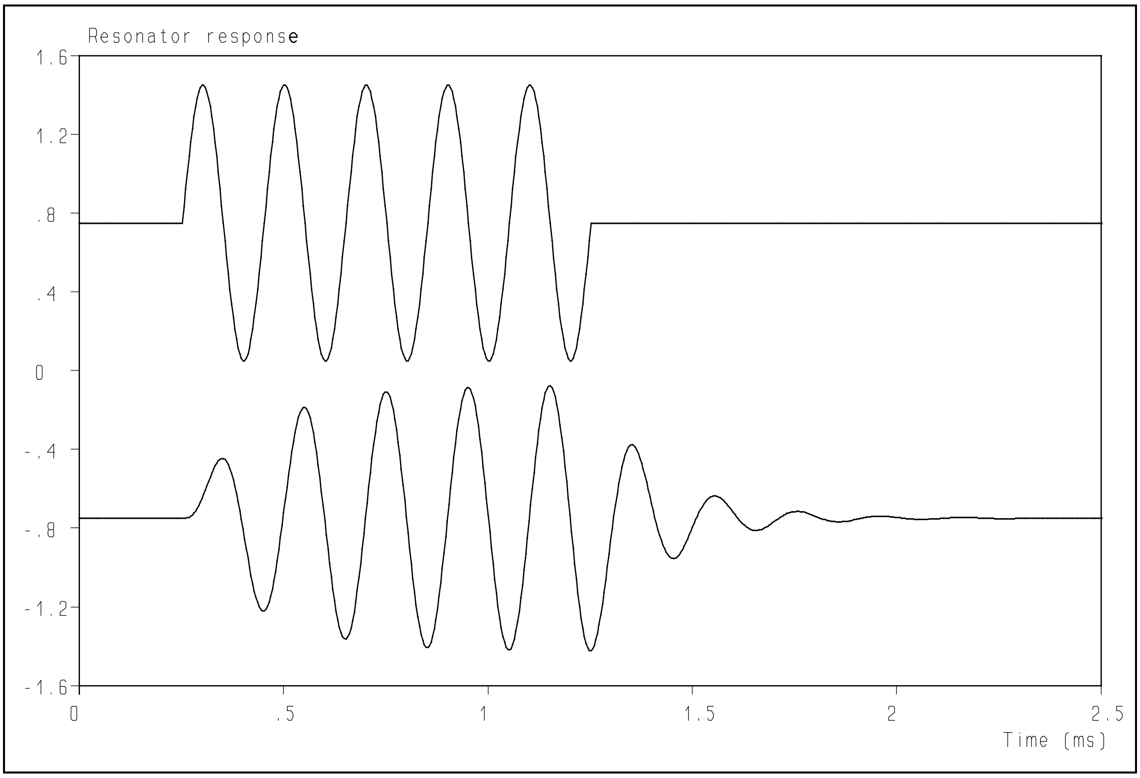


**Figure 35**: Resonators are slow because they need time to start and time to decay.

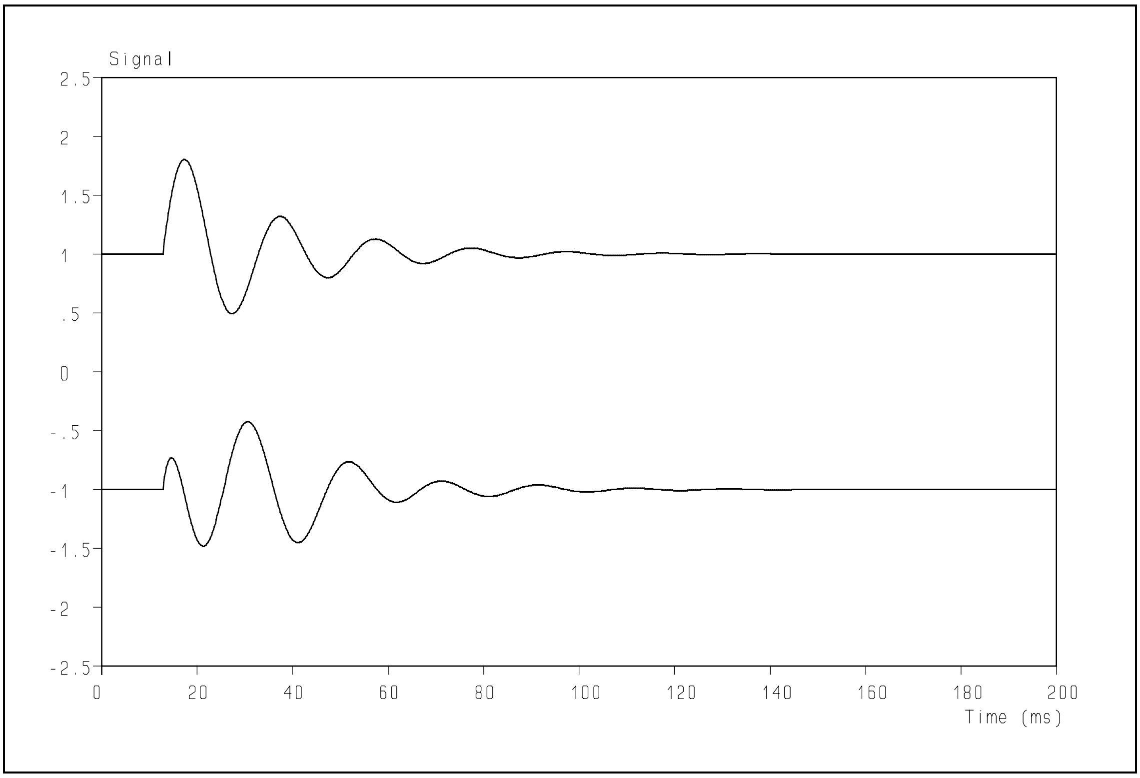


**Figure 36**: Response of a bass-reflex system to a simulated kettle-drum signal.

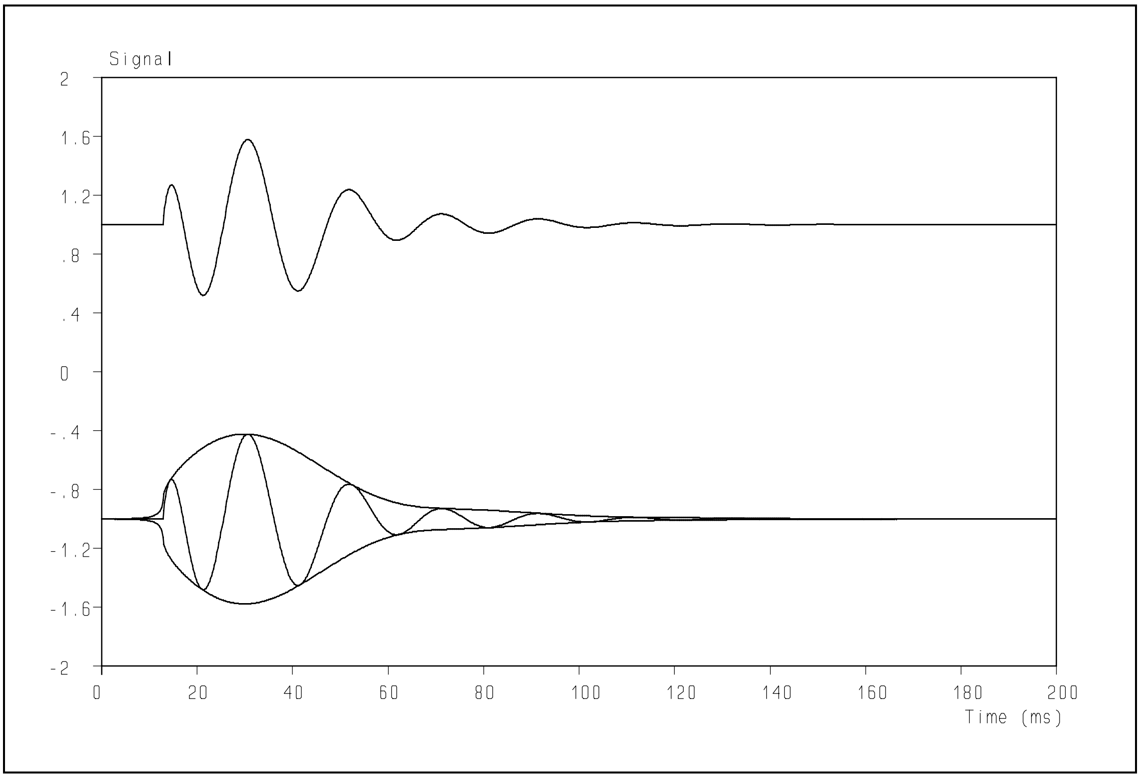


**Figure 37**: Response of a bass-reflex system to a simulated kettle-drum signal showing the envelope as well.

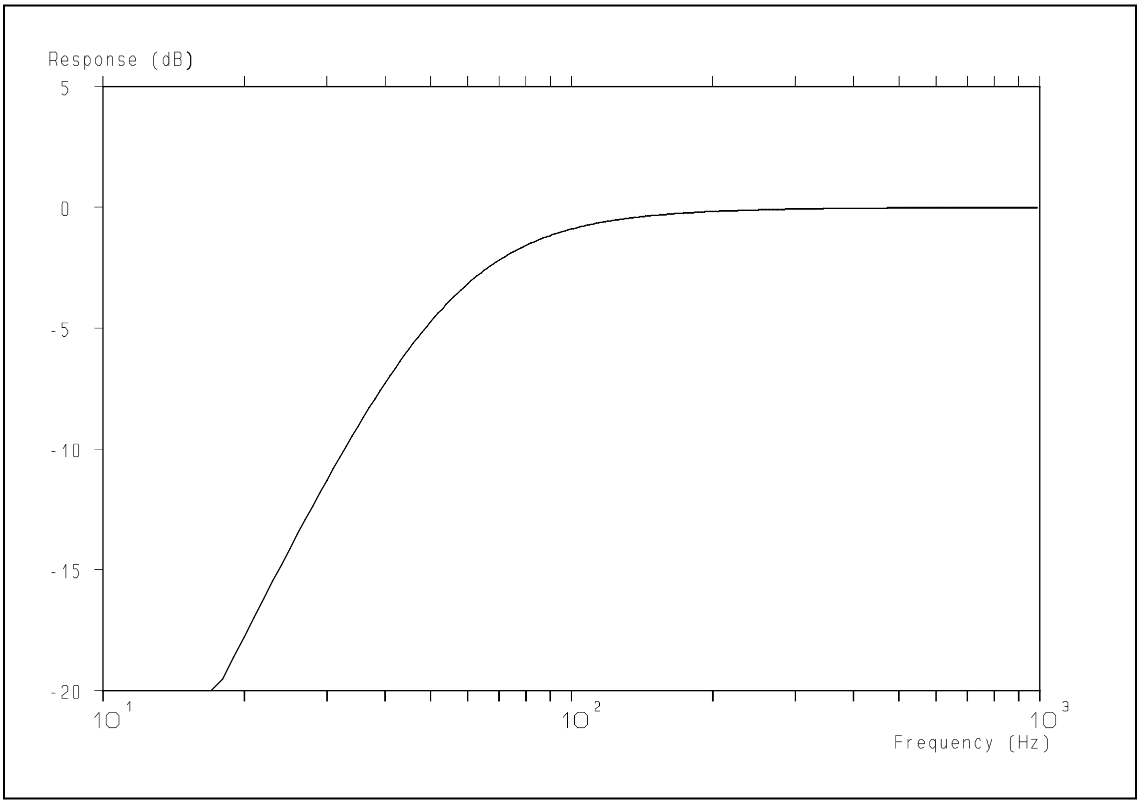


**Figure 38**: Response of the woofer in its acoustic box. Note the absence of a resonance peak, Q approx. 0.63.

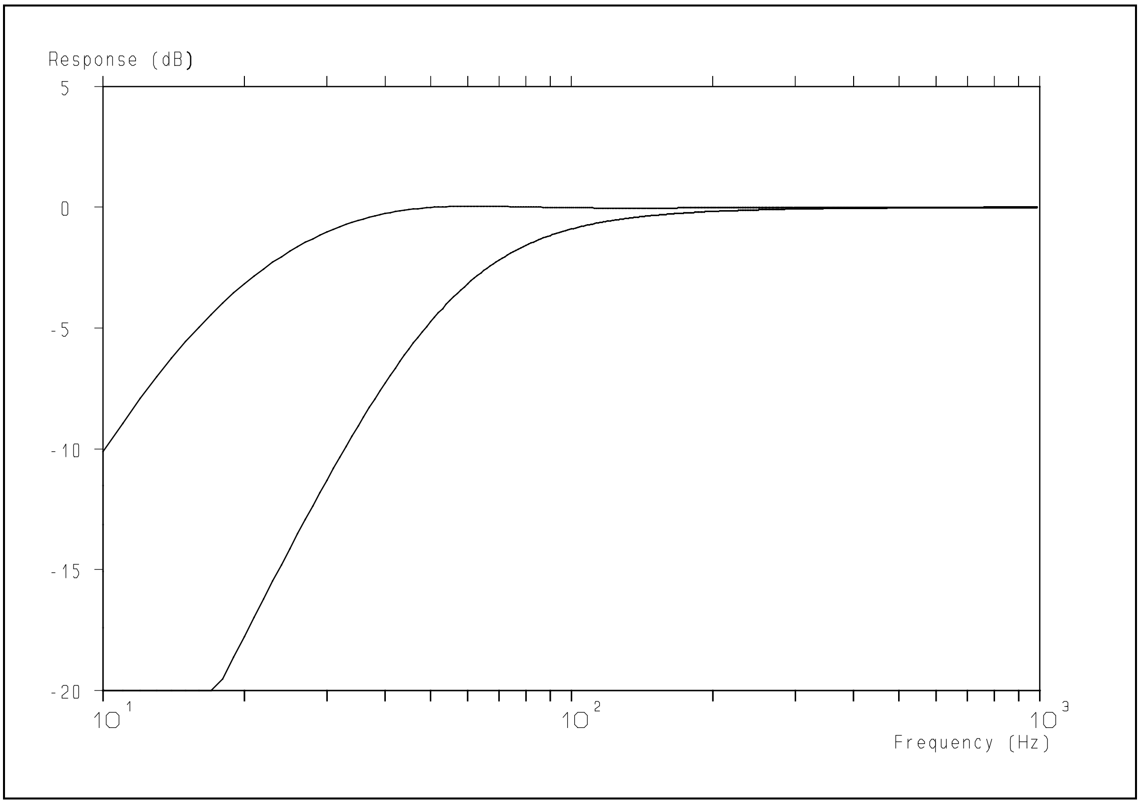


**Figure 39**: Response of the woofer in its acoustic box without (lower curve) and with compensation (upper curve) using the ABC-2.

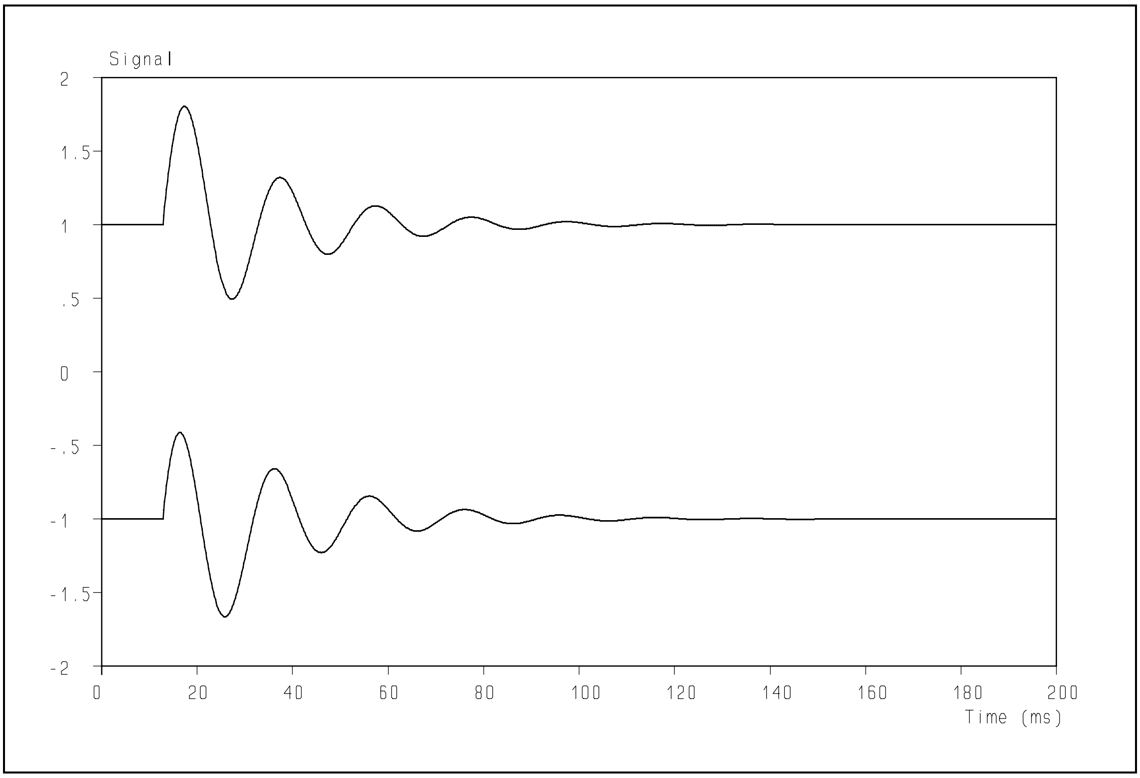


**Figure 40**: Response of the ABC-2 compensated woofer system.

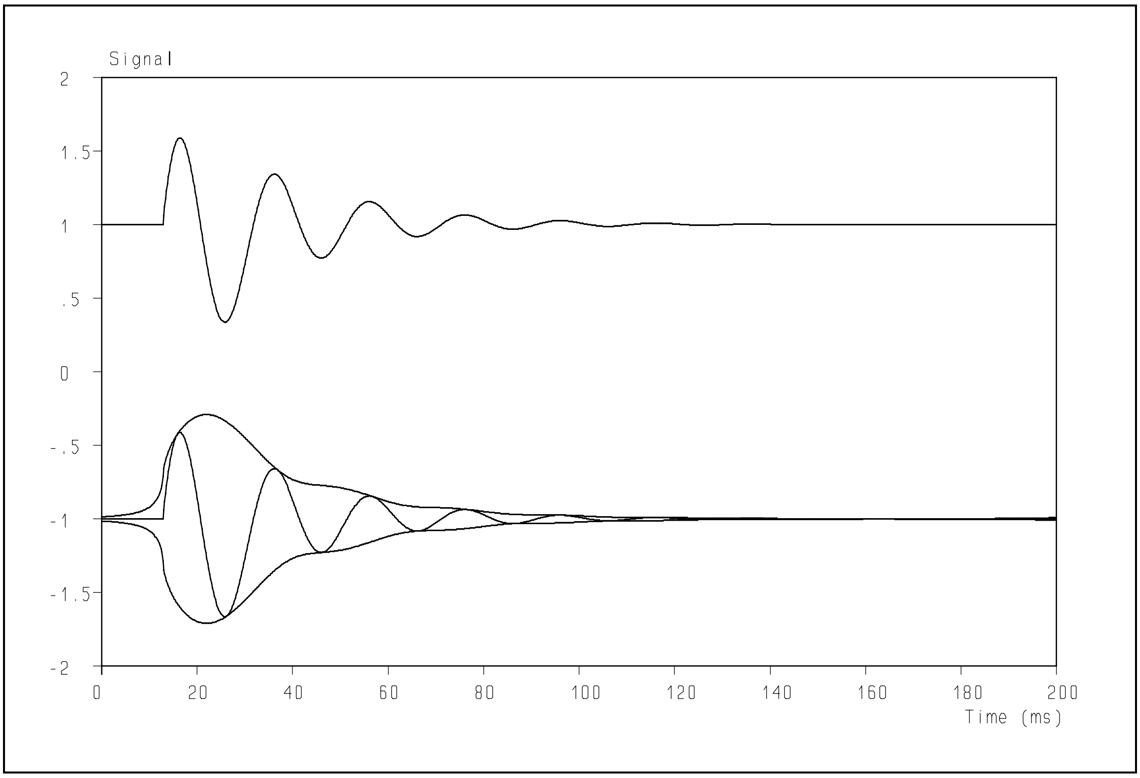


**Figure 41**: Response of an ABC-2 compensated woofer system to a simulated kettle-drum signal showing the envelope as well.

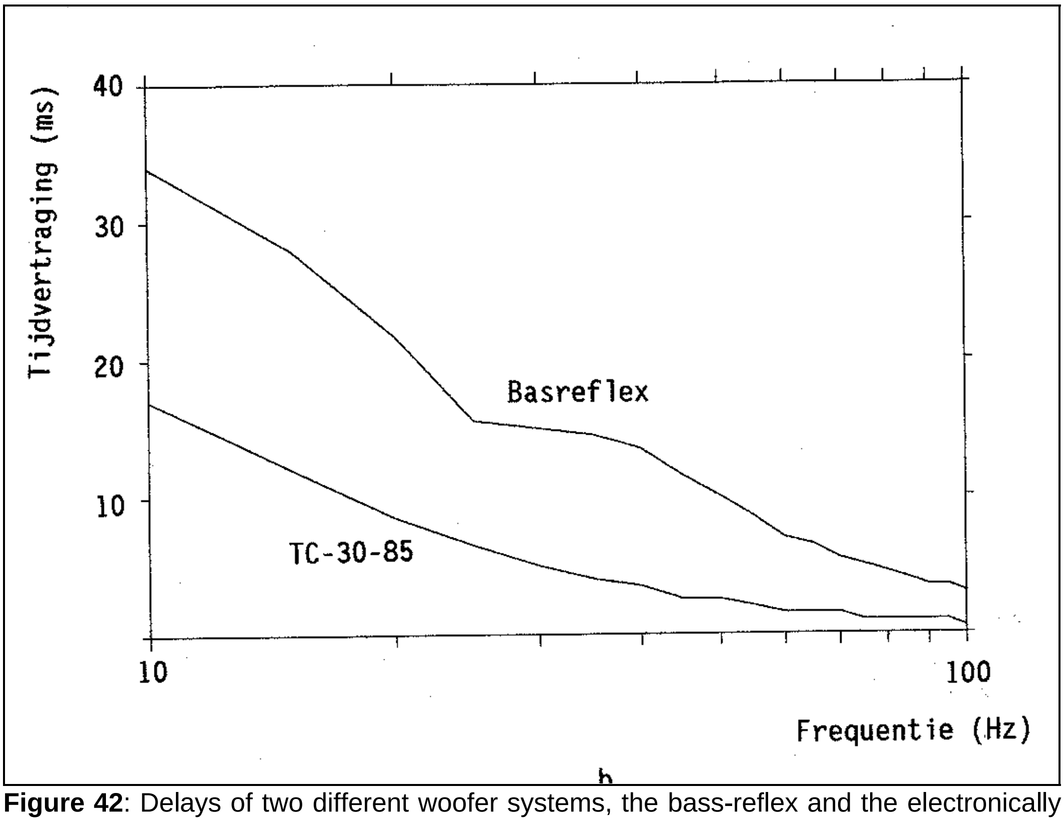


**Figure 42**: Delays of two different woofer systems, the bass-reflex and the electronically compensated woofer in an acoustic box. For more details, see text.

## APPENDIX A

*Recipe to create signals with the same amplitude spectrum*
*but a different temporal characteristic.*

- Define an input signal $f(t)$ as desired (e.g. a two-cycle tone burst).
- Calculate the Fourier Transform of $f(t)$ (= $F(\omega)$).
- Multiply $F(\omega)$ with any desired phase characteristic $\Psi(\omega)$ (e.g. from a sharp analog filter).
- The product is the (complex) spectrum of a different signal with the same amplitude spectrum (= $H(\omega)$).
- Calculate the Inverse Fourier Transformation of $H(\omega)$ to obtain the time signal $h(t)$.

## APPENDIX B

*Recipe to determine the temporal response of signals*
*to systems with any transfer function.*

- Define an input signal $f(t)$ as desired (e.g. a two-cycle tone burst).
- Calculate the Fourier Transform of $f(t)$ (= $F(\omega)$).
- Multiply $F(\omega)$ with the complex transfer function of any desired filter $G(\omega)$ (e.g. from a cross-over filter).
- The product is the (complex) spectrum of the output signal of the filter (= $H(\omega)$).
- Calculate the Inverse Fourier Transformation of $H(\omega)$ to obtain the time signal $h(t)$.

## APPENDIX C

*Simplest numerical implementation of the calculation of the temporal response*
*of signals to systems with any transfer function.*

<u>Fourier Analysis</u>:

```
FOR N% = 0 TO NumberOfSamples% / 2
  SumAn = 0!: SumBn = 0!
  Omega = 2! * Pi * N% / NumberOfSamples% / Dt
  FOR K% = 0 TO NumberOfSamples%
    ActualTime = K% * Dt
    SumAn = SumAn + InputSignal(K%) * COS(Omega * ActualTime)
    SumBn = SumBn + InputSignal(K%) * SIN(Omega * ActualTime)
  NEXT K%
  An(N%) = SumAn / (NumberOfSamples% / 2)
  Bn(N%) = SumBn / (NumberOfSamples% / 2)
NEXT N%
```

<u>Fourier Synthesis</u>:

```
FOR N% = 0 TO NumberOfSamples% / 2
  Omega = 2! * Pi * N% / NumberOfSamples% / Dt
  FOR K% = 0 TO NumberOfSamples%
    ActualTime = K% * Dt
    Response(K%) = Response(K%) + An(N%) * COS(Omega * ActualTime)
    Response(K%) = Response(K%) + Bn(N%) * SIN(Omega * ActualTime)
  NEXT K%
NEXT N%
```

## APPENDIX D

*Listing of a QuickBasic program to calculate the temporal response.*

---

```
' (Inverse) Fast Fourier Transformation using Table Look Up.
'
DECLARE SUB ComplexDiv (Rz1, Iz1, Rz2, Iz2, Rzq, Izq)
DECLARE SUB ComplexMult (Rz1, Iz1, Rz2, Iz2, Rzp, Izp)
DECLARE SUB ComplexVerv (Rz1, Iz1, Rz2, Iz2, Rzv, Izv)
'
OPTION BASE 1
DIM Ftin(1024)          'Input signal
DIM Ftr(1024)           'Real part of input/output signal for FFT
DIM Fti(1024)           'Imaginary part of input/output signal for FFT
DIM Gore(513)           'Real part of transfer function
DIM Goim(513)           'Imaginary part of transfer function
DIM M(25)               'Support array for FFT
DIM Sinus(1280)         'Table of (co)sines for FFT
'
CLS
Pi = 4# * ATN(1#)
'
' Graphic screen definitions:
'
CONST Scherm = 12: CONST Xmin = 1: CONST Ymin = 1
CONST Xmax = 635: CONST Ymax = 478
'
' Initiations for the FFT:
'
N% = 10: Number% = 2 ^ N%: Sdim% = Number% / 2: Sdimh% = Sdim% / 2
'
' Table voor sine and cosine:
'
FOR K% = 1 TO 5 * Number% / 4
  Sinus(K%) = SIN(Pi * (K% - Sdim% - 1) / Sdim%)
NEXT K%
'
' Here is space to put the parameters for the transfer function.
' (to be created by user).
'
' Initialize arrays:
'
FOR K% = 1 TO Number%
  Ftr(K%) = 0!
  Fti(K%) = 0!
NEXT K%
'
Vooraan: CLS : PRINT "Choice from different input signals:"
PRINT
PRINT
PRINT "i = Impulse for low-pass"
PRINT "j = Impulse for high-pass"
PRINT "w = Impulse for Wigner distribution"
PRINT "s = Step"
PRINT "b = Square wave"
```

```
PRINT "t = Tone burst"
PRINT "d = Double sine wave with half cycle silence"
PRINT "p = Kettle drum of 50 Hz."
PRINT "f = Double triangular wave tone burst"
PRINT
'
Signaal$ = ""
DO: Signaal$ = INKEY$: LOOP UNTIL Signaal$ <> ""
'
CLS : GOSUB Signaal                    'A number of signals are "built in"
'
IF Dt = 0 THEN GOTO Vooraan
'
TimeWindow = Number% * Dt
'
' Save the input signal:
'
FOR K% = 1 TO Number%
  Ftin(K%) = Ftr(K%)
NEXT K%
'
' Changeover to graphics screen:
'
CLS : SCREEN Scherm: VIEW (Xmin, Ymin)-(Xmax, Ymax), , 16
WINDOW (1, -1.3)-(Number%, 1.1)
'
' Show the input signal:
'
PRESET (1, Ftr(1))
FOR K% = 1 TO Number%
  LINE -(K%, Ftr(K%)), 14
NEXT K%
'
LOCATE 29, 2: PRINT "Response calculation starting, please wait.";
SLEEP 3
'
Indicator% = -1: GOSUB FFT             'Time ==> Frequency transformation
'
FOR K1% = 1 TO Sdim% + 1
  Frequency = (K1% - 1) / TimeWindow
  Omega = 2! * Pi * Frequency
  '
  ' In this loop, the transfer function should be calculated.
  ' (to be done by user). Use can be made of the subroutines for the
  ' manipulation of complex numbers (see below).
  '
  Gore(K1%) = RealPart: Goim(K1%) = ImaginaryPart
  '
NEXT K1%
'
' Complex multiplication of spectrum and transfer function:
'
FOR K% = 1 TO Sdim% + 1
  CALL ComplexMult(Ftr(K%), Fti(K%), Gore(K%), Goim(K%), RealPart, ImaginaryPart)
  Ftr(K%) = RealPart
  Fti(K%) = ImaginaryPart
```

```
NEXT K%
'
' Fill the high-frequency part of the spectrum with complex conjugate:
'
FOR K% = 2 TO Sdim% + 1
  Ftr(Number% + 2 - K%) = Ftr(K%)
  Fti(Number% + 2 - K%) = -Fti(K%)
NEXT K%
'
Indicator% = -Indicator%: GOSUB FFT     'Frequency ==> Time transformation
'
' Show the original (input) signal and the response:
'
CLS : WINDOW (1, -1.8)-(Number%, 1.6)
'
PRESET (1, Ftin(1) + .5)
FOR K% = 1 TO Number%
  LINE -(K%, Ftin(K%) + .5), 14
NEXT K%
'
PRESET (1, Ftr(1) - .5)
FOR K% = 1 TO Number%
  LINE -(K%, Ftr(K%) - .5), 12
NEXT K%
'
LOCATE 28, 2: PRINT "Press any key to continue."
'
DO: LOOP WHILE INKEY$ = ""
'
SaveToFile: SCREEN 0: CLS
'
PRINT "Do you want to save this result (s) or do you want to quit (q)?"
Bewaar$ = ""
DO: Bewaar$ = INKEY$: LOOP UNTIL Bewaar$ <> ""
'
Bewaar$ = UCASE$(Bewaar$)              'Make the choice case-insensitive
IF Bewaar$ = "Q" THEN CLS : END
IF NOT (Bewaar$) = "S" THEN GOTO SaveToFile
'
INPUT "On which file (without extension) do you want the results to be saved? ", Filename$
'
Filename$ = UCASE$(Filename$)
IF LEN(Filename$) > 8 THEN Filename$ = LEFT$(Filename$, 8)
'
Drive$ = "C:"
Directory$ = "\TC\"
Hulp$ = Drive$ + Directory$ + Filename$ + ".DAT"
'
OPEN Hulp$ FOR OUTPUT AS #1
  '
  PRINT #1, Number%: PRINT #1, TimeWindow
  FOR K% = 1 TO Number%
    Tijd = (K% - 1) * Dt
    PRINT #1, Tijd, Ftin(K%), Ftr(K%)
  NEXT K%
  '
```

```
CLOSE #1
'
PRINT "The data have been saved on  "; Hulp$
'
END
'
'
Signaal: '
'
Dt = 0
'
IF Signaal$ = "i" THEN
  Ftr(Number% / 16) = Number% / 20!
  Dt = .00001
END IF
'
IF Signaal$ = "j" THEN
  Ftr(Number% / 16) = 1!
  Dt = .00001
END IF
'
IF Signaal$ = "w" THEN
  N% = 8: Number% = 2 ^ N%: Sdim% = Number% / 2: Sdimh% = Sdim% / 2
  '
  ' Table for sine and cosine:
  '
  FOR K% = 1 TO 5 * Number% / 4
    Sinus(K%) = SIN(Pi * (K% - Sdim% - 1) / Sdim%)
  NEXT K%
  '
  FOR K% = 1 TO 10
    Ftr(Sdim% + K%) = EXP(-.5 * (K% - 5) ^ 2)
  NEXT K%
  Dt = .001
END IF
'
IF Signaal$ = "s" THEN
  FOR K% = 1 TO Number% / 4
    Ftr(K%) = -1!
  NEXT K%
  FOR K% = Number% / 4 + 1 TO 3 * Number% / 4
    Ftr(K%) = 1!
  NEXT K%
  FOR K% = 3 * Number% / 4 + 1 TO Number%
    Ftr(K%) = -1!
  NEXT K%
  Dt = .00001
END IF
'
IF Signaal$ = "b" THEN
  FOR K% = 0 TO Number% - 16 STEP 16
    FOR L% = 1 TO 8
      Ftr(K% + L%) = -1!
    NEXT L%
    FOR L% = 9 TO 16
      Ftr(K% + L%) = 1!
```

```
    NEXT L%
  NEXT K%
  Dt = .00001
END IF
'
IF Signaal$ = "t" THEN
  INPUT "How many cycles do you want ? ", Perioden
  Perioden = INT(Perioden)
  INPUT "Which frequency do you want ? ", Freq
  INPUT "Which sampling frequency do you want ? ", Samplefreq
  Dt = 1! / Samplefreq
  Length = Perioden * Samplefreq / Freq
  IF Length > Number% - Number% / 16 THEN Length = Number% - Number% / 16
  FOR K% = Number% / 16 TO Number% / 16 + Length
    Ftr(K%) = SIN(2! * Pi * (K% - Number% / 16) * Freq * Dt)
  NEXT K%
END IF
'
IF Signaal$ = "d" THEN
  FOR K% = Number% / 16 TO Number% / 16 + 100
    Ftr(K%) = SIN(2! * Pi * (K% - Number% / 16) / 100!)
  NEXT K%
  FOR K% = Number% / 16 + 150 TO Number% / 16 + 250
    Ftr(K%) = Ftr(K% - 150)
  NEXT K%
  Dt = .00001
END IF
'
IF Signaal$ = "p" THEN
  Dt = .0002: Tauw = .02171: Freq = 50!
  FOR K% = 1 TO Number% - Number% / 16
    Tijd = (K% - 1) * Dt
    Fakt = EXP(-Tijd / Tauw)
    Ftr(Number% / 16 + K%) = SIN(2! * Pi * (K% - 1) * Freq * Dt) * Fakt
  NEXT K%
END IF
'
IF Signaal$ = "f" THEN
  FOR K% = 1 TO 25
    Ftr(K% + Number% / 16) = K% / 25!
  NEXT K%
  FOR K% = 26 TO 75
    Ftr(K% + Number% / 16) = (50! - K%) / 25!
  NEXT K%
  FOR K% = 76 TO 100
    Ftr(K% + Number% / 16) = (K% - 100!) / 25!
  NEXT K%
  FOR K% = Number% / 16 + 1 TO Number% / 16 + 100
    Ftr(K% + 100) = Ftr(K%)
  NEXT K%
  Dt = .00001
END IF
'
IF Signaal$ = "t" THEN RETURN
IF Dt = 0 THEN RETURN
'
```

```
Waarden: '
PRINT USING "Delta t = #####.# usec., Samplefreq. is ###.## kHz."; 1000000! * Dt; .001 / Dt
PRINT
PRINT "Do you want these default values (d) or new values (n) ?"
'
Waarden$ = ""
DO: Waarden$ = INKEY$: LOOP UNTIL Waarden$ <> ""
'
IF Waarden$ = "d" THEN RETURN
'
IF Waarden$ = "n" THEN
  INPUT "Which sampling frequency do you want ? ", Samplefreq
  Dt = 1! / Samplefreq
  RETURN
END IF
'
CLS : GOTO Waarden
'
'
FFT: '
'
FOR I% = 1 TO N%
  M(I%) = 2 ^ (N% - I%)
NEXT I%
'
FOR L% = 1 TO N%
  Nblock% = 2 ^ (L% - 1)
  Lblock% = Number% / Nblock%
  Lbhalf% = Lblock% / 2
  K% = 0
  '
  FOR Iblock% = 1 TO Nblock%
    Indeks% = Indicator% * K% + Sdim% + 1
    Indekc% = Indeks% + Sdimh%
    Wkr = Sinus(Indekc%)
    Wki = Sinus(Indeks%)
    Istart% = Lblock% * (Iblock% - 1)
    '
    FOR I% = 1 TO Lbhalf%
      J% = Istart% + I%: Jh% = J% + Lbhalf%
      CALL ComplexMult(Ftr(Jh%), Fti(Jh%), Wkr, Wki, Qr, Qi)
      Ftr(Jh%) = Ftr(J%) - Qr
      Fti(Jh%) = Fti(J%) - Qi
      Ftr(J%) = Ftr(J%) + Qr
      Fti(J%) = Fti(J%) + Qi
    NEXT I%
    '
    FOR I% = 2 TO N%
      Ii% = I%
      IF K% < M(I%) THEN GOTO Erbij
      K% = K% - M(I%)
    NEXT I%
    '
Erbij: K% = K% + M(Ii%)
  NEXT Iblock%
  '
```

```
NEXT L%
'
K% = 0
FOR J% = 1 TO Number%
  IF K% < J% THEN GOTO Nietwissel
  Holdr = Ftr(J%)
  Holdi = Fti(J%)
  Ftr(J%) = Ftr(K% + 1)
  Fti(J%) = Fti(K% + 1)
  Ftr(K% + 1) = Holdr
  Fti(K% + 1) = Holdi
  '
Nietwissel: FOR I% = 1 TO N%
    Ii% = I%
    IF K% < M(I%) THEN GOTO Erbij2
    K% = K% - M(I%)
  NEXT I%
  '
Erbij2: K% = K% + M(Ii%)
NEXT J%
'
IF Indicator% < 0 THEN GOTO Klaar
FOR I% = 1 TO Number%
  Ftr(I%) = Ftr(I%) / Number%
  Fti(I%) = Fti(I%) / Number%
NEXT I%
'
Klaar: RETURN

' Calculates the quotient of the two complex numbers (Rz1,Iz1) / (Rz2,Iz2)
' The outcome is returned on (Rzq,Izq)

SUB ComplexDiv (Rz1, Iz1, Rz2, Iz2, Rzq, Izq)
noemer = Rz2 ^ 2 + Iz2 ^ 2
Rzq = (Rz1 * Rz2 + Iz1 * Iz2) / noemer
Izq = (Iz1 * Rz2 - Rz1 * Iz2) / noemer
END SUB

' Calculates the product of the two complex numbers (Rz1,Iz1) * (Rz2,Iz2)
' The outcome is returned on (Rzp,Izp)

SUB ComplexMult (Rz1, Iz1, Rz2, Iz2, Rzp, Izp)
Rzp = Rz1 * Rz2 - Iz1 * Iz2: Izp = Rz1 * Iz2 + Iz1 * Rz2
END SUB

' Calculates the substituent of two complex impedances (Rz1,Iz1) and (Rz2,Iz2)
' in parallel. The outcome is returned on (Rzv,Izv)

SUB ComplexVerv (Rz1, Iz1, Rz2, Iz2, Rzv, Izv)
r11 = 1!: i1 = 0!
CALL ComplexDiv(r11, i1, Rz1, Iz1, rv1, iv1)
CALL ComplexDiv(r11, i1, Rz2, Iz2, rv2, iv2)
rv3 = rv1 + rv2: iv3 = iv1 + iv2
CALL ComplexDiv(r11, i1, rv3, iv3, Rzv, Izv)
END SUB
```

## APPENDIX E

*Proof of the correct prediction of the temporal response.*

---

As most people are not familiar with the mathematics behind the Fourier Transformation and its inverse, some are reluctant to accept the predictions, made by programs, as listed in Appendix C and D, of the responses in time domain. It is too much of a "black box". I can understand their hesitation and although it is impossible to present a mathematical proof of correctness of the program, listed in Appendix D, I can illustrate the correct prediction by an example.

In sec. 3.2.6, I discussed the analog filters of higher order and their complements. I use these filters and the original prototype was tested. Luckily, I published the response of the filters as it was recorded on a Polaroid picture of the oscilloscope traces (!) in the original paper (ref. 9). Note that in 1979 this was "state of the art"! I still have a copy of that paper and I scanned it in and I was able to include a negative reproduction of this scan. The predicted response of the filters, using the program as listed in Appendix D, is shown in fig. E-1, whereas the salvaged information from the Polaroid oscilloscope picture is shown in fig. E-2. Although the salvaged information is a bit sketchy, it still makes clear that the predicted and measured responses are as close to identical as can be retrieved from this information.

Although this is, of course, no proof of the correctness of the program, I hope this helps to build trust and confidence in the calculation procedure as laid down in the program, listed in Appendix D.

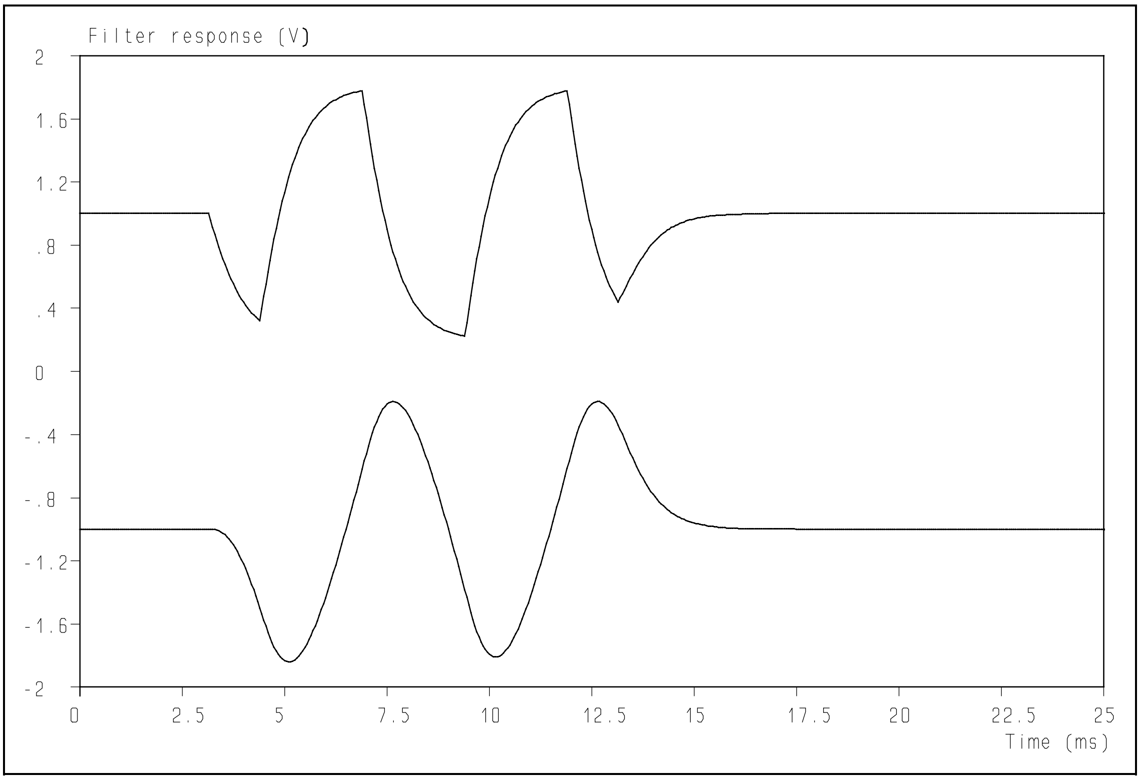


**Figure E-1**: Calculated response of analog filter sections. Compare with fig. D-2.

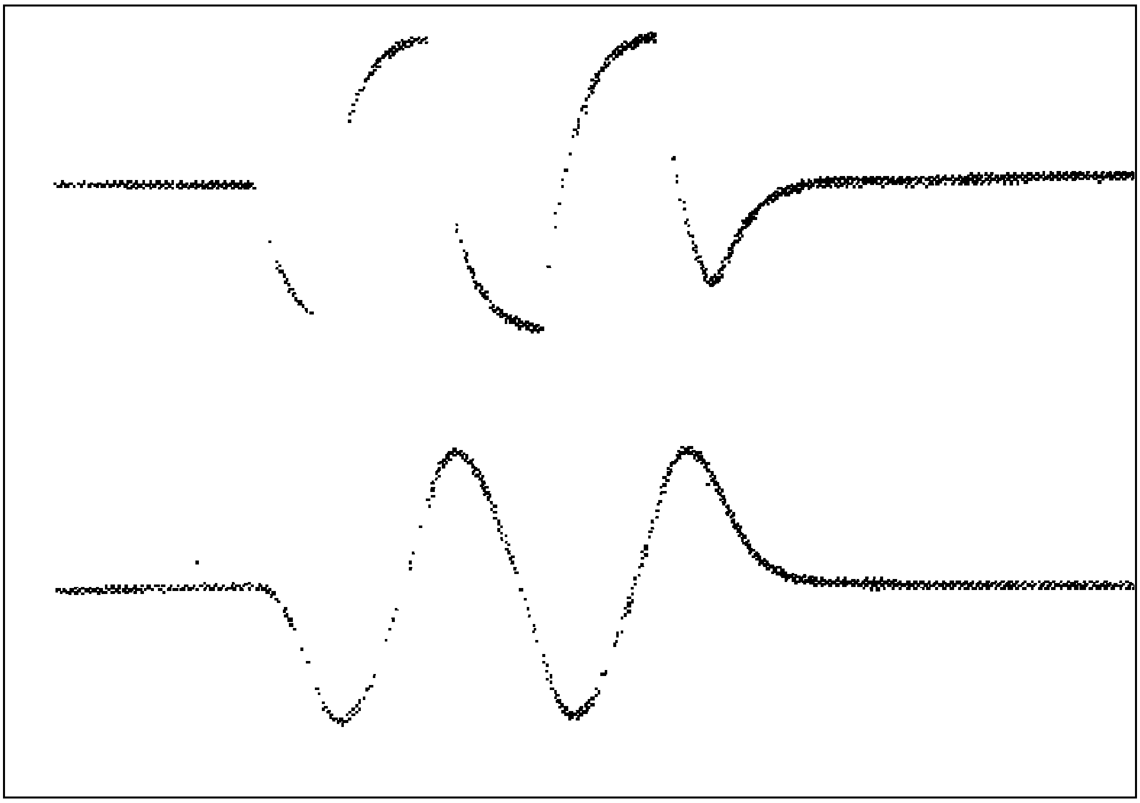

**Figure E-2**: Salvaged negative of polaroid oscilloscope picture, showing the output of the prototype of the analog filters. Compare with fig. D-1.