The influence of the output impedance on a passive cross-over filter

Author: Dr. Hans R.E. van Maanen (Temporal Coherence)

Date of issue: 5 February 2012

The higher output impedance of valve amplifiers is often mentioned when the pro’s and con’s of valve amplifiers are discussed. In general, valve amplifiers have a higher output impedance, and thus a lower damping factor, than semiconductor amplifiers:

$$De = \frac{Z_l}{Z_u}$$

in which:

De	=	Damping factor	
Z_l	=	(Nominal) impedance of the loudspeaker	Ω
Z_u	=	Output impedance of the amplifier	Ω

An often mentioned disadvantage of a low damping factor is coloration, caused by the given that the impedance of a loudspeaker is not a constant, but shows strong fluctuations. Especially the resonances in the low frequency range (of the woofer itself and -by using the base-reflex principle- of the port) are often mentioned because it results in more pronunciation in the low frequency range, but with a somewhat "woolly" character because the amplifier has less control over the movement of the woofer. However, we also often see "humps" in the impedance characteristic around the cross-over frequency between the woofer and the squawker and the cross-over frequency between the squawker and the tweeter. These are the result of an increasing impedance towards the cross-over frequency of the loudspeaker, reproducing the lower frequencies and, similarly, the resonance of the loudspeaker, reproducing the higher frequencies. This also gives rise to coloration. Both phenomena can clearly be seen in fig. 1, in which the measured impedance of a passive filtered base-reflex loudspeaker is shown. The port resonance at approx. 23 Hz and the woofer resonance at approx. 60 Hz can clearly be distinguished as well as the "humps" around the cross-over between the woofer and the squawker (at approx. 500 Hz) and the cross-over between the squawker and the tweeter (at approx. 3500 Hz).

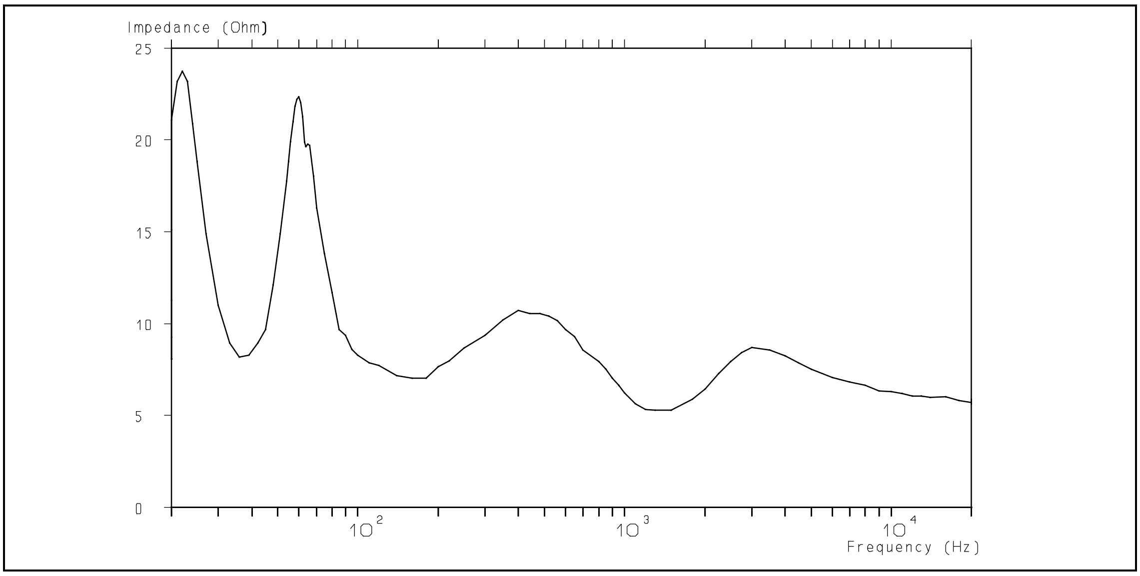

Figure 1: Impedance as measured of a 3-way base-reflex loudspeaker with a passive cross-over filter.

But another effect, which also plays a role, is hardly ever mentioned: the output impedance of the amplifier also influences the time constants of the cross-over filter. And that influence acts on the low-pass exactly opposite as for the high-pass filter. In a simple example: with a low damping factor the tweeter "comes in" already at lower frequencies and the woofer continues to produce up to higher frequencies, which leads to a higher sound pressure level around the cross-over frequency, compared to a high damping factor. This effect is added to the increase due to the higher impedance. In order to estimate an "order of magnitude" we can do the calculation for a simple system. We will look at a 2-way system with first-order filtering. We will assume ideal, perfect loudspeaker units, both concerning the impedance -pure Ohmic and constant- and their frequency response. When we choose the time constants of the low and high pass filters (LPF and HPF) to be identical, we obtain a perfect system with a high damping factor, as can be seen from the following equations:

$$\tau_w = \frac{L}{R_w} \qquad \tau_t = R_t\,C \qquad \tau_w = \tau_t = \tau$$

$$LPF = \frac{1}{1 + j\omega\tau} \qquad HPF = \frac{j\omega\tau}{1 + j\omega\tau}$$

$$LPF + HPF = \frac{1}{1 + j\omega\tau} + \frac{j\omega\tau}{1 + j\omega\tau} = \frac{1 + j\omega\tau}{1 + j\omega\tau} = 1$$

in which:

τ_w = Time constant of the woofer filter s
L = Inductance of the coil of the woofer filter H
R_w = (Ohmic) impedance of the woofer Ω
τ_t = Time constant of the tweeter filter s
R_t = (Ohmic) impedance of the tweeter Ω
C = Capacitance of the capacitor of the tweeter filter F
j = Imaginary operator ($j^2 = -1$)
ω = Radial frequency (= 2π·frequency) rad/s

However, when the output impedance of the amplifier is not zero, but has a value above zero of R_u, something changes:

$$LPF = \frac{R_w}{R_u + j\omega L + R_w} = \left(\frac{R_w}{R_u + R_w}\right)\left(\frac{1}{1 + j\omega\left(\dfrac{L}{R_u + R_w}\right)}\right)$$

$$HPF = \frac{R_t}{R_u + \dfrac{1}{j\omega C} + R_t} = \frac{j\omega R_t C}{1 + j\omega(R_u + R_t)C} = \left(\frac{R_t}{R_u + R_t}\right)\left(\frac{j\omega(R_u + R_t)C}{1 + j\omega(R_u + R_t)C}\right)$$

We see that the time constants in both branches of the filter have been modified. Assuming that the loudspeaker units are pure resistors and that $R_w = R_t$, we can introduce the following substitution:

$$k = \frac{R_w}{R_u + R_w} = \frac{R_t}{R_u + R_t} \quad (< 1)$$

from which follows:

$$LPF = k\left(\frac{1}{1 + j\omega k\tau}\right) \qquad HPF = k\left(\frac{j\omega\frac{\tau}{k}}{1 + j\omega\frac{\tau}{k}}\right)$$

We see that in this case the woofer continues till higher frequencies and that the tweeter starts at lower frequencies. Of course, this has consequences for the summed signal:

$$LPF + HPF = k\left(\frac{1}{1 + j\omega k\tau} + \frac{j\omega\frac{\tau}{k}}{1 + j\omega\frac{\tau}{k}}\right) = k\left(\frac{1 + j\omega\frac{2\ \tau}{k} - \omega^2\tau^2}{1 + j\omega\left(k\tau + \frac{\tau}{k}\right) - \omega^2\tau^2}\right)$$

When we look at the ration between the response at low and high frequencies and the response around the cross-over frequency, (where $\omega^2\tau^2 = 1$) we see that it is equal to:

$$\frac{\frac{2}{k}}{k + \frac{1}{k}} = \frac{2}{1 + k^2}$$

When $k = 1$ (ideal voltage source with an output impedance of 0) there is no increase around the cross-over frequency, but when $k = 0.8$ (output impedance is ¼ of the loudspeaker impedance), there is an increase of 1.72 dB. With a damping factor of 2, the increase is already 2.83 dB. It is evident that this kind of effects are audible with "High-End" systems, apart from the phase errors which additionally are introduced.

It can be concluded that a non-negligible output impedance of the amplifier will have its influence of the properties and the subjective experiences of a loudspeaker with a passive cross-over filter. This is not only about coloration in the low frequency range with a "woolly" character by the limitation of control, but also around the cross-over frequencies coloration can occur and both the phase and the temporal response will be modified. A manufacturer of loudspeakers should actually specify a damping factor or enable an adjustment to the damping factor by the user.

The in this paper discussed problem is one of the many reasons why "Temporal Coherence" systems are equipped with active cross-over filters. But many more good reasons can be mentioned, as described extensively in "The Advantages of "Active" Audio Systems over "Passive" Systems" (6 June 2006) by Dr. Hans R.E. van Maanen, which can be found on www.temporalcoherence.nl next to other interesting papers.