AN EXTENDED MODEL FOR IMPEDANCE AND COMPENSATION OF ELECTRO-DYNAMIC LOUDSPEAKER UNITS AND AN ALGORITHM FOR THEIR DETERMINATION.

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Abstract.

The impedance of loudspeaker units can be described by lumped-element models, which increase the efficiency and accuracy of cross-over design if applicable over the entire frequency range. We have extended current models to obtain excellent description in the entire useful range. Also, algorithms have been designed which automatically find the best parameters for the lumped-element model and a compensation network, which are easily implemented on a PC.

Summary.

Most lumped-element models, used for the description of the impedance of electro-dynamic loudspeaker units deviate from the true behaviour in a part of the useful frequency range. However, the design of cross-over networks can be done more efficiently and accurately using computer models, in which the loudspeaker units are replaced by their lumped-element equivalents. The accuracy of the design thus depends on the accuracy of the description of the impedance by the lumped-element model. We have therefore improved such models to describe the entire useful frequency range in order to eliminate the problem of deviation.

Our final model comprises eight elements, and therefore requires automatic determination by an efficient algorithm. The major problem with such algorithms is that as the number of dimensions increases, the risk of arriving at local or sub-minima increases more than proportional with the number of dimensions. This is caused by the fact that functional surfaces become very complicated in higher-dimensional spaces and easily "wormholes" can be created. It is therefore essential to find initial parameter values, which are sufficiently close to the final values such that converging towards the global minimum can be guaranteed. This has been achieved because the choice of the initial values for the parameters is integrated into the algorithm. The optimization algorithm chosen was a vectored minimum search method, adapted for this application, with a four step 10-fold decrease in parameter variation.

A similar algorithm has been used to determine the values of a network, which compensates the variations of the impedance of a loudspeaker unit. This impedance compensation network is designed in such a way that the phase after compensation is minimised, so that the resulting impedance is as close to Ohmic as possible. The application of such networks reduces the problems in the design of cross-over networks for passive loudspeaker cross-over networks or reduces the amplifier-loudspeaker unit interactions in active loudspeaker systems. Both cases yield a better, more musical, sound of the loudspeaker unit.

In the paper the model and the algorithms will be described and how they have been implemented on a PC. Several results of the application to real, but different, loudspeaker units will be presented, showing that excellent agreement has been obtained over the entire useful frequency range. Future work is directed towards a fully automatic measurement of the loudspeaker unit, followed by the design of the compensation network.

1. Introduction.

The electrical impedance of a loudspeaker unit is of importance for both the design of the cross-over filter and impedance compensation networks. In either case the magnitude and phase of the units' electrical impedance needs to be known. The measured values can be used, but for the design of the above mentioned networks it is more attractive to describe the loudspeaker unit in familiar terms of passive components (coils, capacitors and resistors). Therefore in literature many lumped-element models can be found which describe the impedance behaviour of a loudspeaker unit (some examples can be found in ref. 1 - 3). (A fortunate aspect is that the radiation of sound is only of secondary importance). A much used model is shown in figure 1. However, such models give significant differences between the actual and predicted impedance at higher frequencies. This is caused by the assumption that the voice coil behaves electrically as a real coil, which is, in practice, too simplistic. One should also keep in mind that the impedance behaviour -especially the phase- is influenced by the interaction between the resonance and the voice coil behaviour and that incorrect description will lead to incorrect design of the filters. We have therefore extended the model by introducing a more complicated network for the voice coil. This will be described in sec. 2, followed by the description of a search algorithm to calculate the parameters of the model on the basis of measured data. Then some results will be presented and the possibilities for application to the design of impedance compensation networks, in which the same search algorithm is used, will be discussed.

2. Extension of the model.

The resonance of the loudspeaker unit is usually adequately described by the common models. The raise of the impedance at higher frequencies is, however, predicted too high as well as the phase angle between voltage and current. This leads to an incorrect description of the interaction between resonance, voice coil and cross-over filter. The incorrect prediction of the voice coil impedance is caused by the negligence of the eddy current damping -very often deliberately introduced to improve the impedance behaviour of the loudspeaker unit- which causes a deviation of the behaviour of the voice coil from an inductor. We have therefore modified the voice coil inductance in the more complicated network of figure 2. The voice coil is separated into four sub-coils with an -arbitrarily chosen- ratio of 1 : 0.5 : 0.25 : 0.25. Parallel to three sub-coils are resistors, representing the eddy current damping. These three resistors are the additional three parameters that can be chosen to fit the measured curve. In this way the influence of the eddy current can be made frequency dependent, which is a new aspect compared to previous models.

3. Search algorithm.

The optimisation of the eight parameters, describing the electrical impedance of the loudspeaker, is too complicated to do by hand. Therefore the optimisation is realised using a search algorithm. The application of the algorithm requires three prerequisites:

1. An estimation of initial values of the parameters.
2. The definition of variance of the parameters.
3. The definition of an error function which quantises the deviation of the fitted from the measured curve.

Ad 1.
The estimation of the initial values of the parameters is important, because in this problem we are applying a minimum search in a 9-dimensional space. If the initial values are too far off, there is a high probability that a local minimum is found and not the global minimum. For a 2-dimensional case this is illustrated in figure 3. In a 9-dimensional space, the number of local minima is usually much higher and so-called "wormholes" can lead the search algorithm to erroneous results. A good estimate of the initial values is therefore of prime importance. This will be therefore be done by the algorithm software itself.

Ad 2.
To optimise the parameter a small variation is applied. If the variation results in a lower error value, it is saved, else it is ignored. We have chosen to start with a variation of 2 % for each parameter. During the optimisation this variation is four times reduced with a factor of 10, so the final variations are 2.10^{-6} of the value.

Ad 3.
The error function must be chosen in such a way that two important requirements are fulfilled:

1. A better fit should result in a lower error value.
2. The most important parts of the fit should be weighted accordingly.

Ad 1.
Two options that are used very often are the squared difference error value and the absolute difference error value. These can be expressed as:

$$E.V. = \sum_{i=1}^{N} W_i \,.\, (P_i - M_i)^2 \qquad (1)$$

$$E.V. = \sum_{i=1}^{N} W_i \,.\, ABS(P_i - M_i) \qquad (2)$$

in which:
E.V. = error value.
W_i = weighting factor for value at frequency i.
P_i = predicted value by model at frequency i.
M_i = measured value at frequency i.
ABS = absolute value of argument.

The impedance of the loudspeaker unit, predicted by the model, has a smooth behaviour. The actual measured impedance curve can show narrow-band irregularities (caused e.g. by partial resonances in the cone). These irregularities can give a large contribution to the error value using (1), which is, of course, a disadvantage of this error function. The error function (2) is less sensitive for this effect. However, this effect can be also be suppressed by reducing the weighting values of the frequencies where these irregularities show up. In our experience the best results have been obtained with (1). However, one should keep in mind that any error function, fulfilling the above mentioned requirements, can be used.

The actual algorithm is derived and adapted from one described in ref. 4. It searches for the minimum of the error function, which is a function of the parameters, by variation of these parameters. The algorithm follows a number of steps. It uses a direction vector which has as many components as there are parameters. This vector points into the direction of the minimum and during the execution of the algorithm it is constantly modified. For a 3-dimensional case this is illustrated in figure 4. At the start of the algorithm all the components of this vector are put to zero, as at that moment the direction in which the minimum must be found is not yet known. The algoritm continues along the following steps:

1. Set the number of times the variance of the parameters is to be reduced (this was chosen to be four times in this case, each time with a factor of 10.)

For each **component** of the direction vector:

2. Increase (in a absolute sense) the direction vector component by +1 or -1 (depending on the sign of the vector component). Modify the parameter with the direction vector component times the variance and calculate the error value[1]. If the error value is lower than the previously determined minimal value, keep these values for the direction vector component, parameter and error value and continue with the next component. If not, go to step 3.

3. Decrease (in an absolute sense) the direction component by +1 or -1 (depending on the sign of the vector component). Modify the parameter with the direction vector component times the variance and calculate the error value. If this is lower, save these values for the direction vector component, parameter and error value and continue with the next component. If not, go to step 4.

4. Put this direction vector component to 0 (zero) and continue with the next component.

Repeat the steps 2 - 4 until all direction components are 0 (zero). This corresponds to the point which is less than a single variance step from the minimum. To determine the position better, the variance must therefore be reduced:

5. If not all variance reduction steps have been used, reduce the variances of all the components (by a factor of 10 in our case) and go to step 2.

At the end of the procedure, the minimum of the error function has been found, in this case meaning that variation of the current values of the parameters with a single step of the variations (which are in the order of 2.10^{-6}.) do not result in a lower value of the error value. One should keep in mind that further reduction of the variations might lead to problems: the accuracy of the registration of floating point variables in most computer languages is 6 - 7 digits, so smaller variations do not lead to a change of the value of the parameter. Therefore double precision should be used in those cases. However, in our view this is not really necessary, as the spread in the measurements, different loudspeaker units and electrical components do not allow for more accurate results.

4. Results for the impedance curve fitting.

The parameters used for the description of the electrical impedance and their initial settings are:

1. The DC resistance (is estimated from the impedance at 10 Hz).
2. The voice coil inductance (is calculated from the impedance at the highest frequency measured).
3. The eddy current damping of the first section of the voice coil (is set at 0.8 times the value of the estimated DC resistance (parameter 1)).
4. The eddy current damping of the second section of the voice coil (is set equal to the value of the estimated DC resistance (parameter 1)).
5. The eddy current damping of the third section of the voice coil (is set at twice the value of the estimated DC resistance (parameter 1)).
6. The damping of the resonance (is set at the difference between the impedance at the resonance frequency minus the estimated DC resistance (parameter 1)).
7. The resonance frequency of the loudspeaker unit (is estimated from the peak in the impedance curve at the lowest frequency).
8. The Q-factor of the resonance (is set at 2.5).

In some of the initial settings the measured data are directly used, in others "intelligent guesses" are used, in which knowledge of common practice is used. In general these estimates lead to the global minimum of the error function value. The algorithm thus finds the optimum value of the parameters. This

[1] Note that the attempted change of the parameter is in most cases more than the value of the variation, because the direction vector component is different from 1. In this way the algorithm efficiently finds its way towards the minimum in increasingly larger steps, until the step takes it over the minimum.

is illustrated in figure 5, in which the measured and fitted impedance curve are shown in the upper trace and the predicted phase behaviour in the lower trace. We have been able to use this algorithm on many different loudspeaker units in all sizes of acoustic boxes and we have obtained an excellent fit for all cases. We are therefore convinced that the model leads to good results, moreover because it is based on physics.

5. Impedance compensation.

The basic reason for the development of the model for the electrical impedance and the automatic parameter optimisation was to design impedance compensation networks for these loudspeaker units[2]. There are two important reasons for the application of impedance compensation on loudspeaker units:

1. An Ohmic impedance behaves better in a passive cross-over network.
2. An Ohmic load is an easier load for the power amplifier.

Ad 1.
The prediction of a passive cross-over filter on Ohmic loads is much easier to understand than that of a complex load. (As a side remark one can note that to describe the behaviour of a passive cross-over network the electrical behaviour of the loudspeaker unit is essential, so this technique can there be used as well).

Ad 2.
If the load for the amplifier is not Ohmic, this results in offsets in the feedback circuitry of the power amplifier: it still has to deliver current when the voltage is already zero and vice versa. Because the amplifier is not designed for such conditions, this can lead, depending on the design, to an increase of the distortions of the amplifier compared to the Ohmic load[3]. Listening experiments with our listening panel have indeed revealed that the "musicality" of the amplifier increases considerably if the load is Ohmic.

In order to limit the amount of components we have chosen for a five component impedance compensation network. This consists of a RC network which should compensate the increase of the impedance at higher frequencies due to the voice coil inductance and a LRC network, which should compensate the resonance of the loudspeaker unit. This is illustrated in figure 6. As these networks interact with each other and with the loudspeaker unit, it is close to impossible to design the impedance compensation network by hand. Therefore a similar optimisation has been used to find the optimum values for the components of the impedance compensation network. The criterion used is to minimise the **phase** between voltage and current as much as possible over the entire frequency range of interest.

Here we again encounter the problem of the choice of the initial values for the components. These are derived from the values, found in the optimisation of the model for the loudspeaker impedance:

If we define τ = Lvc2 / Rdc, then C1 = τ / Rdc and R1 = Rdc. The resistor R2 is chosen such that the parallel value of R2 and the maximum value of the impedance at resonance equals Rdc. The values of L and C2 are derived from the resonance frequency (which is set equal to the resonance frequency of the loudspeaker) and the -from experience chosen- Q-factor of 1.5. These are subsequently varied in a similar way as the parameters which describe the loudspeaker impedance. It is, however, a simpler problem because it is limited to a 6-dimensional space. As a consequence it is less sensitive to

[2] We mean with impedance compensation networks a network, which is placed in parallel to the loudspeaker unit. The resulting impedance of the network and the loudspeaker unit in parallel should than be as close to Ohmic as possible, so the magnitude should be independent of frequency and it should have zero degree phase angle between voltage and current for all frequencies.

[3] Note that almost all power amplifiers are specified for an Ohmic load.

"wormholes" and the like. Therefore we have found no problems in the optimisation of these compensation networks using such initial values as described above.

6. Results of the impedance compensation.

The application of the optimisation procedure to the loudspeaker of figure 5 leads to the compensation network of figure 7. The application of such a compensation network leads to the impedance curve shown in figure 8. In practice the results are -of course- influenced by the tolerances of the components, yet the results are surprisingly good. We have measured impedance variations of smaller than ± 0.25 Ω over a frequency range from 20 - 2000 Hz.

7. Conclusions.

The described model for an electro-dynamic loudspeaker fits the experimental data very well up to high frequencies, sufficiently far above the cut-off frequency of the cross-over filter. Therefore the impedance is described accurately by this model and it can be used for the correct design of a cross-over filter as well as for the design of an impedance compensation network.

The 8 parameters of this model for the description of the loudspeaker units' impedance are found by a minimum search algorithm that converges correctly, provided the intial values of the parameters are chosen close to their final values. This choice is done by the software, depending on the measured values, thus correct convergence of the algorithm can be guaranteed. The parameter and actual values for the impedance compensation network, which has also been described in this paper, are obtained by a similar algorithm. Using this software package the impedance of the loudspeaker unit can be approximated by a resistor automatically. This improves the interaction between power amplifier and loudspeaker unit. It also improves the behaviour of a passive cross-over filter. Measurements of compensated units have shown that this can be done with great accuracy.

8. Future extensions.

We intend to extend this work into a fully automatic procedure in which the impedance measurements are performed by the computer as well and by the introduction of the Fourier transform in order to measure the phase behaviour as well, so as to describe the loudspeaker unit even more accurate.

References:

1. G.A. Briggs, "Loudspeakers", Fifth edition (1958), Rank Wharfedale Limited.

2. R. Römer and G. Schwamkrug, "Luidsprekers, fabels en feiten", Second edition, Elektuur B.V.

3. J.N. Moreno, "Measurement of Loudspeaker Parameters Using a Laser Velocity Transducer and Two-Channel FFT Analysis", AES Journal, Volume 39, Number 4 (April 1991), pp. 243 - 249.

4. P.R. Adby and M.A.H. Dempster, "Introduction to Optimization Methods", Chapman and Hall, London (1974).

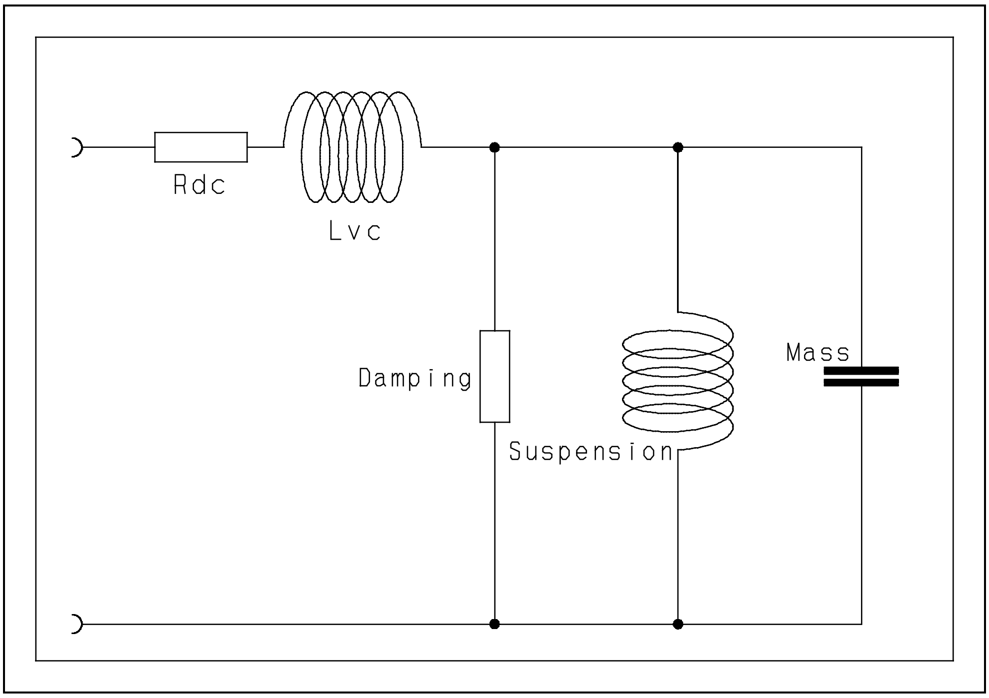

Figure 1: Electrical equivalent of loudspeaker impedance. Rdc is DC resistance, Lvc is inductance of voice coil.

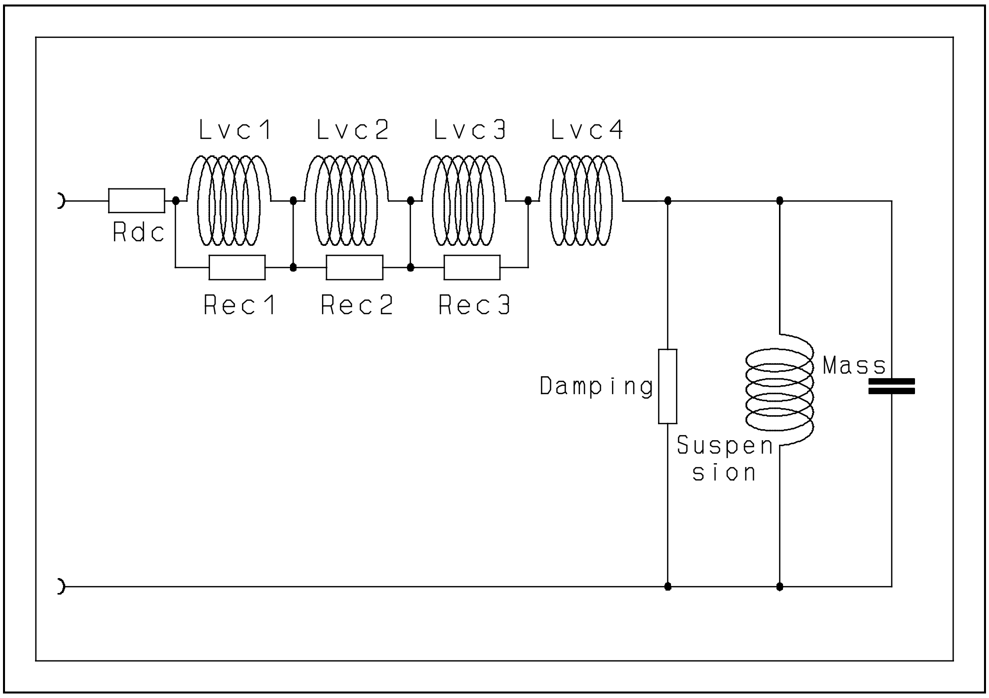

Figure 2: Extended lumped-element model of the electrical impedance of a loudspeaker unit. Rec is eddy current damping.

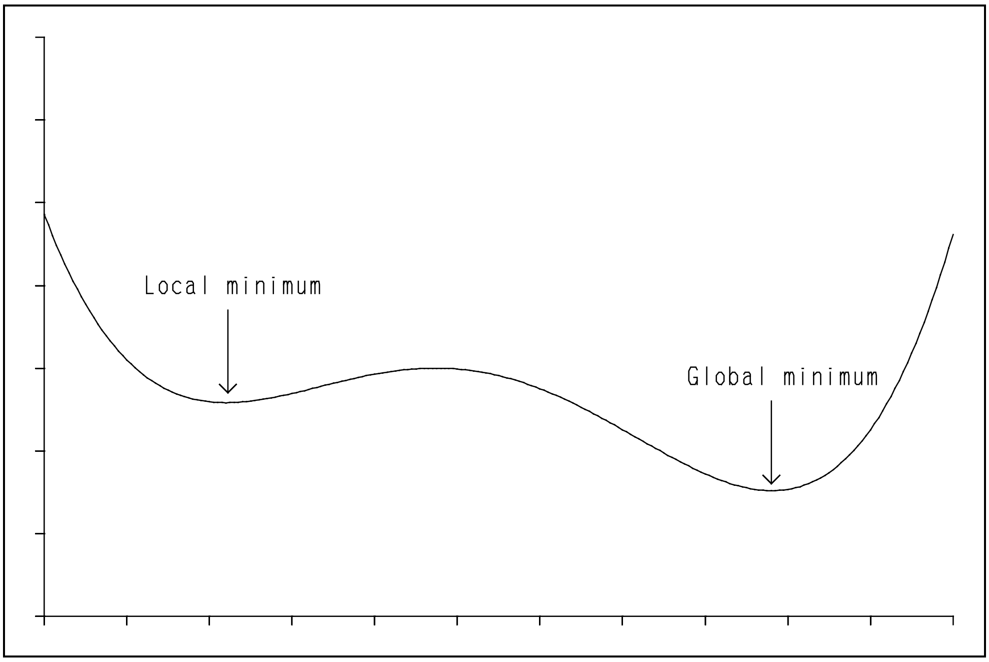

Figure 3: Illustration of local and global minimum in a two-dimensional case. If the initial values are too far to the left, a minimum search algorithm will find the local and not the global minimum.

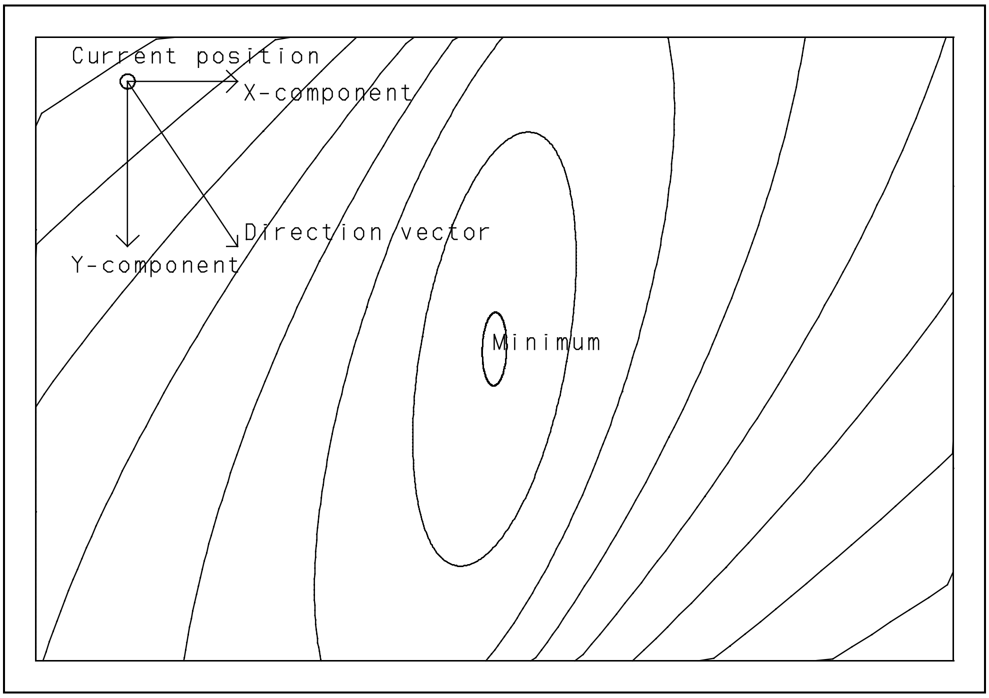

Figure 4: The direction vector in a two-dimensional case. Note that the continuous optimisation adapts the vector to the local slope.

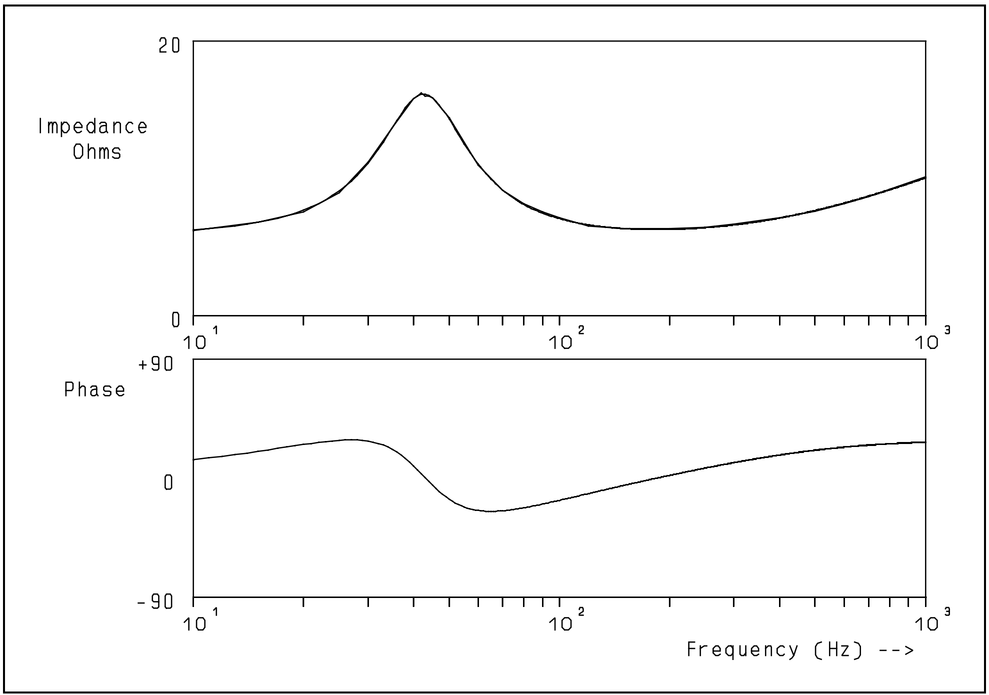

Figure 5: Result of the curve fit. Upper trace shows measured and fitted curve, lower trace the predicted phase, based on the model.

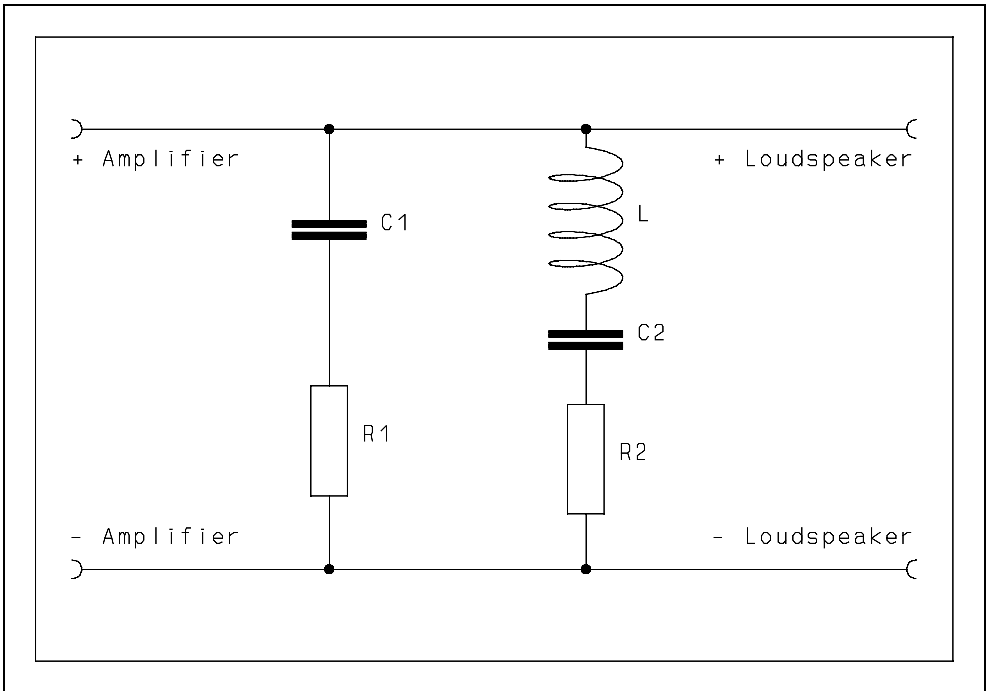

Figure 6: Impedance compensation network used. RC network is to compensate voice coil inductance, LRC to compensate loudspeaker resonance.

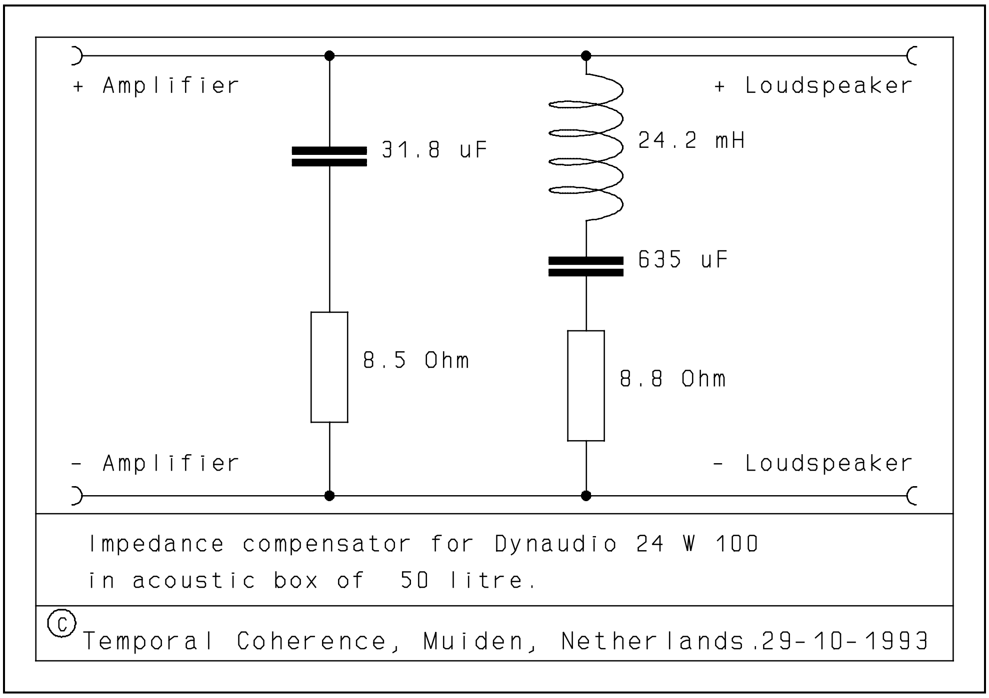

Figure 7: Impedance compensation network used to obtain the results presented in fig. 8.

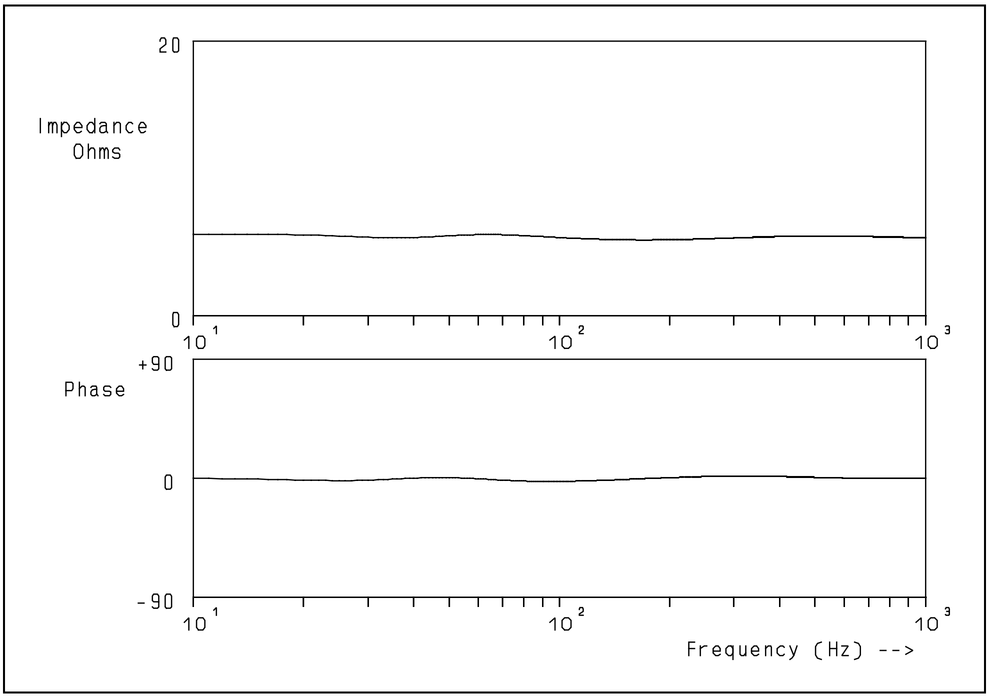

Figure 8: Impedance behaviour of loudspeaker unit with the impedance compensation network in parallel. Compare with fig. 5.