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Non-linear distortions in capacitors

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ABSTRACT

Many people have claimed that capacitors have a notable influence on the audible quality of systems. We have identified one of the major causes of non-linear distortions in capacitors. Charging the capacitor will result in an attractive force acting on the conducting plates. As no material is infinitely stiff, this force will reduce the thickness of the dielectricum and thus *increase* the capacitance. This process occurs in both phases of an AC signal in the same way and is thus non-linear. In this paper the consequences of this process are discussed. It should be noted that other passive components like resistors and inductors can also show similar non-linear behaviour.

1. INTRODUCTION

A capacitor consists of two conducting plates, separated by an insulating layer. The capacitance is proportional to the surface area of the plates, inversely proportional to the thickness of the insulating layer (often referred to as "dielectricum") and proportional to the polarizability of the dielectricum. But contrary to what people might think, the capacitance is *not* a constant, it depends on the charge on the capacitor. Charging the capacitor will result in an attractive force acting on the conducting plates. As no material is infinitely stiff, this force will reduce the thickness of the dielectricum and thus *increase* the capacitance. The force is always attractive, reversing the polarity will result in the same attractive force. This phenomenon will therefore lead to non-linear distortion; with an AC signal, odd harmonics are created and when the AC signal is superimposed on a DC voltage, it will mostly be even harmonics. It will be obvious that the more flexible the insulating layer is, the larger the distortion will be. For the quasi-static case (signal frequency << mechanical resonance frequency, see below) the phenomenon is illustrated in fig. 1.

We have derived a differential equation which describes this phenomenon. We will present the derivation of this equation, which, unfortunately, cannot be solved analytically because it is non-linear. Therefore, we have solved this equation numerically. We have verified the properties of the numerical solver by applying it to a linear differential quation (which can be solved analytically) and compared the results. This showed that the numerical solution gives very accurate results as the differences were only minor.

The mechanical properties of the capacitor are modelled as a resonator and the properties of the resonator (resonance frequency, *Q*-factor and damping) are of prime importance on the non-linear behaviour of the capacitor. This means that the audible quality of capacitors can be influenced by several different properties like the elasticity modulus of the dielectricum, its mechanical damping and the effective moving mass of the conducting plates.

We will show that these kind of effects can lead to severe intermodulation distortion in passive cross-over filters as applied in loudspeakers.

2. THE DIFFRENTIAL EQUATION

Suppose a capacitor **C** consisting of two plates each with a surface area **A** with a distance **d** between the plates and a dielectrical material with relative dielectrical constant ε_r between the plates, with $\varepsilon_0 = 8.854\ 10^{-12}\ \ F/m$. Then the capacitance of this capacitor is given by:

$$C = \frac{\varepsilon_0\, \varepsilon_r\, A}{d} \qquad \text{-1-}$$

Suppose a momentary potential difference $\mathbf{V_{C,t}}$ between both plates. This voltage will create an always attractive force $\mathbf{F_t}$ between the plates, given by the "Walker" equation:

$$F = \frac{\varepsilon_0\, \varepsilon_r\, A\, V^2}{2\, d^2} \qquad \text{-2-}$$

Now suppose that the distance between the plates is not a constant, but varies because the stiffness of the plates and the dielectrical material is not infinitely large, but equal to a certain value, given by the stiffness constant $\mathbf{K_{el}}$. As a consequence of the force and the stiffness a mass **m** of the capacitor starts to move and the distance $\mathbf{x_t}$ of the plates will change, where $\mathbf{x_0}$ equals the plate distance when no charge is present on the plates. Also there will be some damping of the movement, given by the damping constant $\mathbf{K_f}$. The total differential equation of this damped oscillator is given by:

$$\frac{\varepsilon_0\, \varepsilon_r\, A\, V_{c,t}^2}{2\, x_t^2} = m\, \frac{d^2 x_t}{d\, t^2} + K_f\, \frac{dx_t}{dt} + K_{el}\,(x_t - x_0) \qquad \text{-3-}$$

In the above equation it is assumed that there is only one unique stiffness constant and only one unique damping constant. In reality there will be more than one. In that case the total differential equation will have a right hand side equal to the summation of all the partial masses with their damping and stiffness. However for ease of calculation and understanding, we will continue this paper using equation 3.

From now on we will name the capacitor $\mathbf{C_t}$ with non constant distance between the plates a "swinging capacitor", and in practice it will function as a "microphonic capacitor".

3. FIRST ORDER FILTER EQUATIONS

Now consider a first order high pass filter situation, as is e.g. used as a part of a cross-over filter in a loudspeaker. Consider the momentary capacitance to be $\mathbf{C_t}$, the driving alternating voltage with frequency **f** of the source to be $\mathbf{Vc_t}$, the output impedance of the voltage source to be $\mathbf{R_S}$, while the capacitor is sending its AC-voltage into a load with resistance $\mathbf{R_L}$ **>>** $\mathbf{R_S}$. The current $\mathbf{I_t}$ through this series circuit is given by $\delta\mathbf{Q_t}/\delta\mathbf{t}$, where $\mathbf{Q_t}$ is the momentary electrical charge on each capacitor plate. Knowing that $\mathbf{Q_t = C_t \cdot Vc_t}$ the actual voltage over the capacitor in relation to the driving voltage $\mathbf{Vs_t}$ is given by equation 4:

$$Vc_t = Vs_t - (R_s + R_l)\left(C_t\, \frac{\partial Vc_t}{\partial t} + V_t\, \frac{\partial C_t}{\partial t} \right) \qquad \text{-4-}$$

Equation 5 shows the relation between the change in time of the capacitance related to the changing distance $\mathbf{x_t}$.

$$\frac{\partial C_t}{\partial t} = -\, \frac{\varepsilon_0\, \varepsilon_r\, A}{x_t^{\,2}}\, \frac{\partial x_t}{\partial t} \qquad \text{-5-}$$

Using equations 5 and 6, the actual voltage $\mathbf{VL_t}$ over the load resistance can be calculated and is given by equation 6.

$$VL_t = (Vs_t + Vc_t)\left(\frac{R_L}{R_S + R_L} \right) \qquad \text{-6-}$$

4. NUMERICAL SOLUTIONS

With the differential equation 3 and the electrical circuit equations 4 to 6 a set of differential equations is created which needs to be solved. However, the total system is non-linear, so a standard analytical solution is not applicable. Therefore we developed a numerical approach, which is discussed below.

Because of space limitations, we will only give a course outline of the numerical solver. It is a well-established solving technique using time discretisation and no voltage and current discretisations. The pitfall is that the time discretisation introduces delays between the different derivatives (first and second order) and thus a deviation between the analytical and numerical differential equations which are solved. The obvious solution is to use very small time steps, but this can lead to a large volume of calculations and –far more sneaky- to round-off errors in the changes of the signal to be numerically integrated. This will lead to erroneous results. Finding the correct balance between the two conflicting requirements is part of the solver and this has been verified by applying the solver to a linear second order differential equation (which can be solved analytically) and subsequently comparing the analytical and numerical solutions. The results of this verification are presented in figures 2 and 3 and it is clear that the results are indistinguishable. The results of the solver can be trusted such that it will provide the consequences of the non-linear effects as the model introduces.

5. RESULTS AND DISCUSSION

When a sine wave is sent through the high-pass filter as mentioned above, the value of the swinging capacitor will vary with the voltage across the capacitor. As a consequence, the current, flowing through the load resistor $\mathbf{R_L}$ will not be a pure sine wave, but will be –relatively speaking- larger when the voltage across the capacitor is larger. An extreme case, in which the plate distance variation equals 3.5 %, was chosen to illustrate the effect as the distortion in the current is visible without further analysis. It also shows that the phase of the distortion products is influenced by the phase across the capacitor. Note that the "cut-off" frequency is no longer a constant as the capacitance is not a constant!

When an AC signal is applied to such a filter, the distortion products are –as a first approximation- proportional to the square of the input voltage. This means that the effects increase strongly with the signal strength. As it is likely that there will be a relation with the maximum voltage a capacitor can handle. the effects are likely to be strong in passive cross-over loudspeaker filters where strong signals are used.

6. INTERMODULATION IN CROSS-OVER FILTERS

The non-linear properties as outlined above become apparent in cross-over filters where mostly large AC signals are applied in combination with large capacitors (both in capacitance and size) because of the large currents which flow through it, compared to e.g. the electronics in a control amplifier. Such capacitors will –in general- show the non-linear behaviour clearly and it is therefore not really surprising that cross-over filters are often mentioned when the audible properties of capacitors are discussed. Therefore a typical case was modelled: two pure sine waves were used as input signal. The frequency of the high frequency was roughly equal to the cut-off frequency of the high-pass filter, the frequency of the low frequency component is 1/8 of the frequency of the high frequency component and its amplitude is 4 times the amplitude of the high frequency component. The initial input frequencies can be identified at frequencies "1" and "8" in figure 5. Note that the amplitude of the low-frequency signal has been reduced as can be expected by the action of a high-pass filter. The results, presented in fig. 5, can be called stunning. Although the calculation is based on the model as presented above, the clear presence of a large number of intermodulation products with a significant signal strength is worrying to say the least. It is obvious that such effects can clearly introduce audible differences between different types of capacitors. But now that we have unveiled the underlying cause of the problem, an objective way to determine the audible properties of different types of capacitors can be developed. The choice of the dielectricum and the way the capacitor is constructed can now also be optimised.

The results, presented in figs 4 and 5 are obtained at frequencies an order of magnitude below the mechanical resonance frequency of the capacitor. When the frequency of the input signal is (far) above the resonance frequency of the capacitor, the movement of the plates cannot follow the input signal and therefore the effects will be small, compared to those below the resonance frequency. The higher the ratio of the input frequency and the resonance frequency, the smaller the effects are.

When the frequency of the input signal is close to the resonance frequency, the movement of the plates is enhanced and so are the effects. Predicting these effect is, however, severely hampered by the lack of knowledge of the properties of this resonance, especially the *Q*-factor and therefore it is –at this moment- not opportune to present results as these could be severely different from reality. This will require experiments to determine the properties of the resonance. However, as it is not unlikely that especially capacitors, used for passive cross-over filters, will resonate in the audio-band, the choice of the capacitor and the cut-off frequency can greatly influence the audible quality.

Capacitors, used in electronics, like in amplifiers and active cross-over filters, often have a DC component across the plates. This will change the properties of the distortion products: under such conditions, the distortion products will be mainly even harmonics, albeit that in this case the distortion is proportional to the AC signal strength as long as the peak-to-peak value of the AC component is smaller than the DC voltage. But as the signals are usually smaller than in passive cross-over filters, the distortions will be smaller as well. Still, the design of such electronics can be optimised with the properties of the capacitors in mind.

Other passive components like resistors and inductors also show non-linear behaviour. In a resistor, the momentary, instantaneous, dissipation is proportional to the square of the current. The dissipation will increase the temperature and –in general- also the resistance of the resistor. This results in similar phenomena as described for the capacitor, but in reality, the effect will be minor because of the thermal inertia of the resistor: the variations in the dissipation will –in many cases- be too fast to change the temperature of the material from which the resistor is made notably. An exception could be when low frequencies are involved and / or with thin film resistors.

Inductors can show non-linear effects in two ways: first of all when a core is present due to the non-linear relation between the magnetic properties of the core and the magnetic field, and secondly because of forces, acting on the coil as the current flowing through the inductor will change its physical dimensions, which is, similar as in the case of a capacitor, invariant for inverting the sign of the current. Both will influence the inductance in a non-linear way and thus create distortions, similarly as occur in capacitors, as described in this article.

7. CONCLUSIONS

We have discovered a major cause of non-linear distortion in capacitors and we have shown that this effect can create significant –and thus audible- distortions in sound signals. Especially in passive cross-over filters this effect can introduce high levels of intermodulation distortion and thus lead to audible differences between capacitors.

The underlying causes of these distortions have now been unveiled and therefore, an objective way to determine the audible quality of capacitors can be developed. This is a major step forward, compared to endless listening tests with often incomprehensible results. Also, the design of electronics can be optimised with the non-linear properties of capacitors as parameter.

8. REFERENCES

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[2] **Menno van der Veen**: "Low level audio signal transfer through transformers conflicts with permeability behavior inside their cores", 122nd AES convention 2007, paper 7125

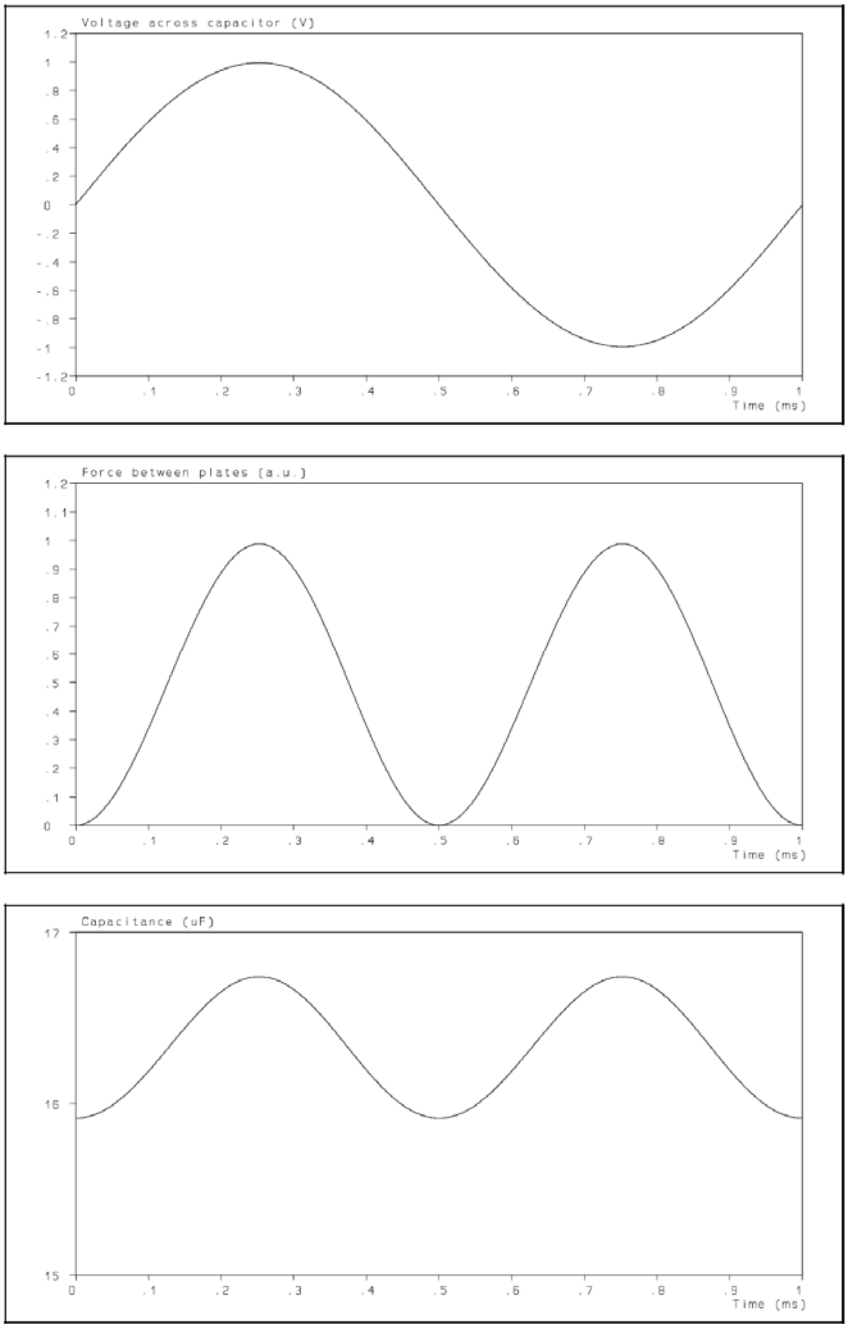

Figure 1: The basic phenomenon which creates the non-linear effect in capacitors

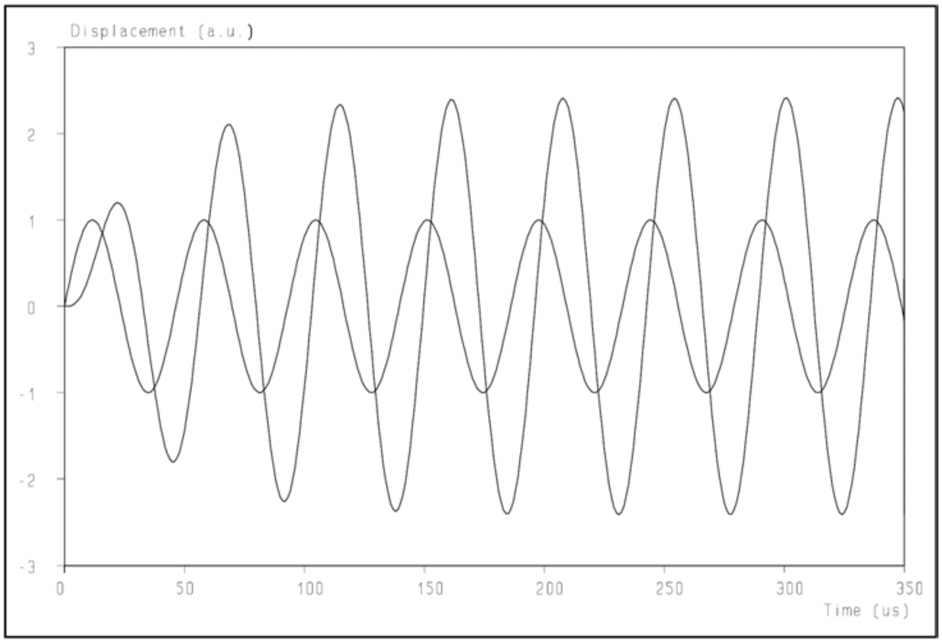

Figure 2: The response of a linear mechanical resonator to a sine wave, which starts at t = 0. Analytical solution of the differential equation.

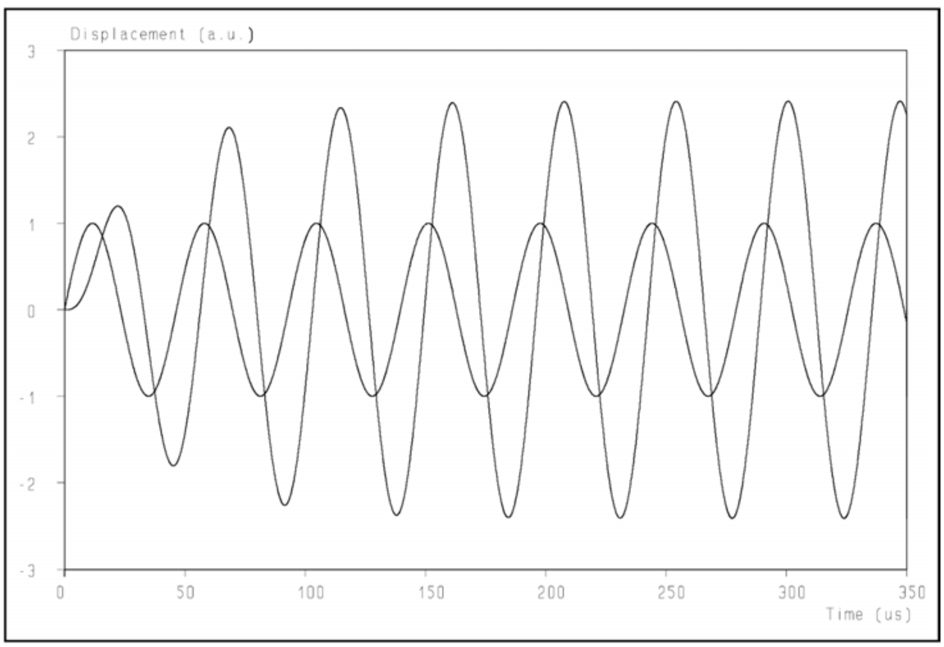

Figure 3: The response of a linear mechanical resonator to a sine wave, which starts at t = 0. Numerical solution of the differential equation.

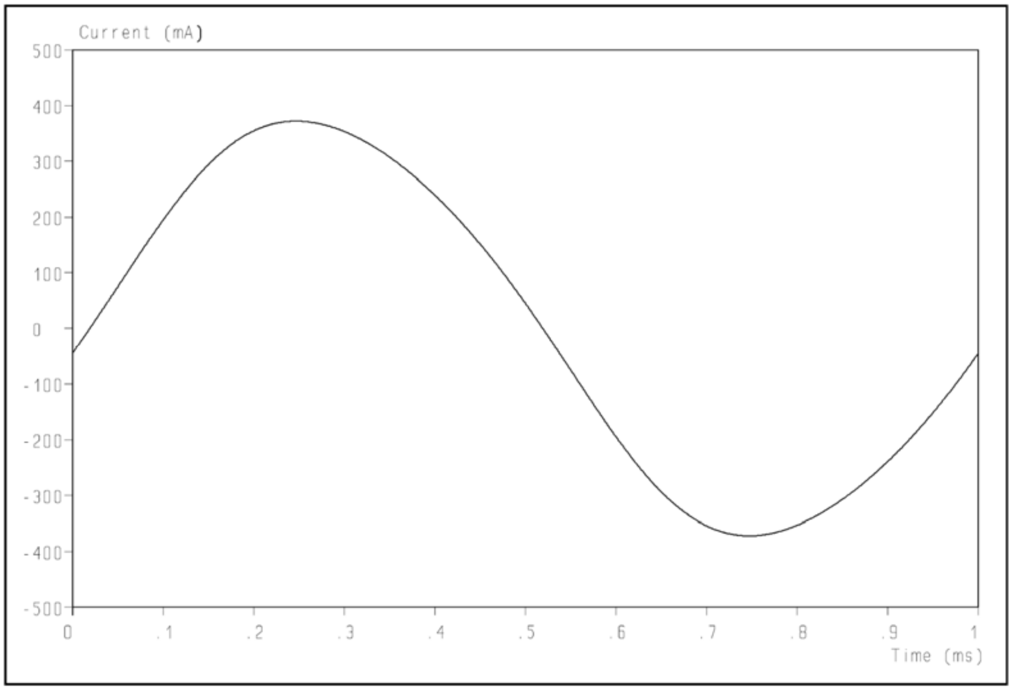

Figure 4: The current, flowing out of the capacitor is no longer a pure sine wave because of the changing of the capacitance due to the voltage across the capacitor.

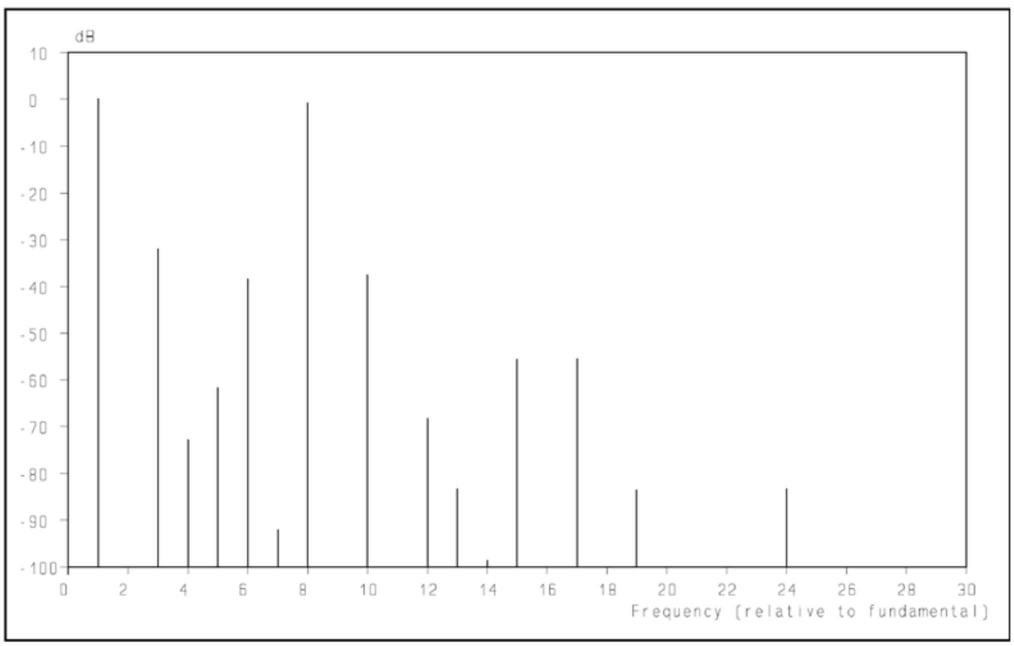

Figure 5: The spectrum of the output signal, clearly showing the intermodulation products. For more details, see text.